MA 110 — Precalculus
Exam 4

Spring 2016
2 May 2016

Name: ______________________________

Section: __________

Last 4 digits of student ID #: __________

This exam has eleven multiple choice questions (five points each) and five free response questions (nine points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		55
12		9
13		9
14		9
15		9
16		9
Webassign Score		100

Record the correct answer to the following problem on the front page of this exam.

(1) Solve for x

$$\sqrt{x-1}+1=x$$

A) $x=1$ and $x=0$

B) $x=1$ only.

C) $x=1$ and $x=2$

D) $x=0$ only.

E) The equation has no solution.

$$\left(\sqrt{x-1}\right)^2=(x-1)^2$$

$$x-1=x^2-2x+1$$

$$0=x^2-3x+2$$

$$0=(x-2)(x-1)$$

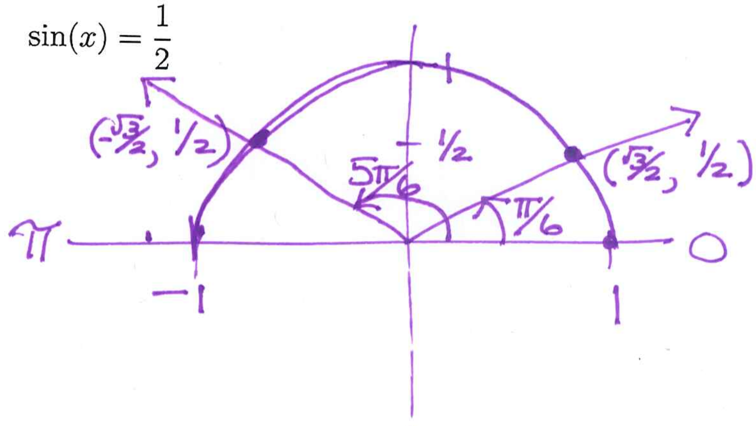
$$x-2=0 \qquad x-1=0$$

$$\boxed{x=2 \qquad x=1}$$

*remember to check answers since you squared both sides

(2) Find all solutions in the interval $[0,\pi)$ for

$$\sin(x)=\frac{1}{2}$$



A) $\frac{\pi}{6}$ and $\frac{-\pi}{6}$

B) $\frac{\pi}{6}$ only.

C) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D) $\frac{\pi}{3}$ only.

E) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(3) Describe the end behavior as $x \to \infty$ of

$$f(x)=\frac{x^3-x}{1+x}$$

A) $y \to 0$

B) $y \to -\infty$

C) $y \to 1$

D) $y \to \infty$

E) $y \to -1$

$$f(x)=\frac{x(x^2-1)}{1+x}$$

$$=\frac{x\cancel{(x+1)}(x-1)}{\cancel{(1+x)}}$$

$$=x(x-1) \text{ where } x\neq -1$$

$$=x^2-x \text{ where } x\neq -1$$

even degree & positive leading coefficient

$x\to-\infty$ $\qquad$ $x\to\infty$

Record the correct answer to the following problem on the front page of this exam.

(4) Find the inverse $f^{-1}(x)$ for

$$f(x) = 5e^{(2x+4)}$$

A) $f^{-1}(x) = \ln(5(2x+4))$

B) $f^{-1}(x) = \dfrac{\ln(\frac{x}{5}) - 4}{2}$

C) $f^{-1}(x) = \dfrac{1}{5e^{(2x+4)}}$

D) $f^{-1}(x) = \dfrac{e^{-(2x+4)}}{5}$

E) $f(x)$ has no inverse since $f(x)$ is not 1-1.

$y = 5e^{2x+4}$

$x = 5e^{2y+4}$

$\frac{x}{5} = e^{2y+4}$

$\ln(\frac{x}{5}) = 2y+4$

$\ln(\frac{x}{5}) - 4 = 2y$

$\frac{\ln(\frac{x}{5}) - 4}{2} = y$

$f^{-1}(x)$

(5) Find the domain of

$$f(x) = \log(x+2)$$

A) $(-\infty, -2)$

B) $(-\infty, 2)$

C) $(0, \infty)$

D) All real numbers.

E) $(-2, \infty)$

$x + 2 > 0$

$x > -2$

(6) If $\sin(\theta) = \frac{5}{7}$ and $\tan(\cancel{x}\ \theta) < 0$, find $\cos(\theta)$

A) $\frac{2}{7}$

B) $\frac{-2\sqrt{6}}{7}$

C) $\frac{2\sqrt{6}}{7}$

D) $\frac{-2}{7}$

E) $\frac{5}{2\sqrt{6}}$

$\sin(\theta) > 0$ & $\tan(\theta) < 0$

only in Quad. II

thus, $\cos(\theta) < 0$

$\cos(\theta) = -\sqrt{1 - \sin^2\theta}$

$= -\sqrt{1 - (\frac{5}{7})^2}$

$= -\sqrt{49/49 - 25/49}$

$= \frac{-\sqrt{24}}{7} = \frac{-2\sqrt{6}}{7}$

Record the correct answer to the following problem on the front page of this exam.

(7) Find the equation of the line passing through $(2, -4)$ and perpendicular to the line $2x - 3y + 2 = 0$.

$-3y = -2x - 2$

$y = \frac{2}{3}x + \frac{2}{3}$

$\perp$ slope $= -\frac{3}{2}$

A) $y = \frac{2}{3}x - \frac{16}{3}$

B) $y = \frac{-3}{2}x - 1$

C) $y = \frac{2}{3}x + \frac{2}{3}$

D) $y = \frac{-1}{2}x - 3$

E) $y = 2x - 8$

$y + 4 = -\frac{3}{2}(x - 2)$

$y = -\frac{3}{2}x + 3 - 4$

$y = -\frac{3}{2}x - 1$

(8) Convert $\left(5, \frac{2\pi}{3}\right)$ in polar coordinates to Cartesian coordinates.

polar $(r, \theta) \Rightarrow$ Cartesian (x, y)

$x = r\cos(\theta)$ $\quad$ $y = r\sin(\theta)$

$x = 5\cos(2\pi/3)$ $\quad$ $y = 5\sin(2\pi/3)$

$x = 5(-1/2)$ $\quad$ $y = 5(\sqrt{3}/2)$

$x = -5/2$ $\quad$ $y = 5\sqrt{3}/2$

A) $\left(\frac{5\sqrt{3}}{2}, \frac{-5}{2}\right)$

B) $\left(\frac{-5}{2}, \frac{10}{\sqrt{3}}\right)$

C) $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

D) $\left(\frac{-5}{2}, \frac{5\sqrt{3}}{2}\right)$

E) $\left(\frac{5}{2}, \frac{10}{\sqrt{3}}\right)$

(9) Find the center of the circle given by

$$x^2 + y^2 + 4x - 6y + 4 = 4$$

A) $(-4, 6)$

B) $(-2, -3)$

C) $(-2, 3)$

D) $(-4, -9)$

E) The equation is not a circle.

$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = \cancel{4} - \cancel{4} + \underline{4} + \underline{9}$

$(x + 2)^2 + (y - 3)^2 = 13$

$(x - (-2))^2 + (y - 3)^2 = 13$

"h" $\quad$ "k"

Record the correct answer to the following problem on the front page of this exam.

(10) Find $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$\sin^{-1}\left(\sin(2\pi/3)\right)$

A) $\frac{2\pi}{3}$

B) $\frac{-2\pi}{3}$

$\sin^{-1}\left(\sqrt{3}/2\right)$

C) $\frac{\pi}{6}$

D) The expression is undefined.

$\pi/3$

E) $\frac{\pi}{3}$

(11) Find $(f \circ g)(x)$ for $f(x) = 2x + 3$ and $g(x) = x^2 + 3x$.

A) $x^2 + 5x + 3$

B) $4x^2 + 18x + 18$

C) $2x^3 - x^2 + 9x$

D) $2x^2 + 6x + 3$

E) $4x^3 + 9x$

$f(g(x))$

$f(x^2 + 3x)$

$2(x^2 + 3x) + 3$

$2x^2 + 6x + 3$

(12) Given

$f(x) = A\sin(bt + c)$ $\quad A = -4,\ b = 3,\ c = -\pi/2$

$$f(x) = -4\sin(3t - \pi/2)$$

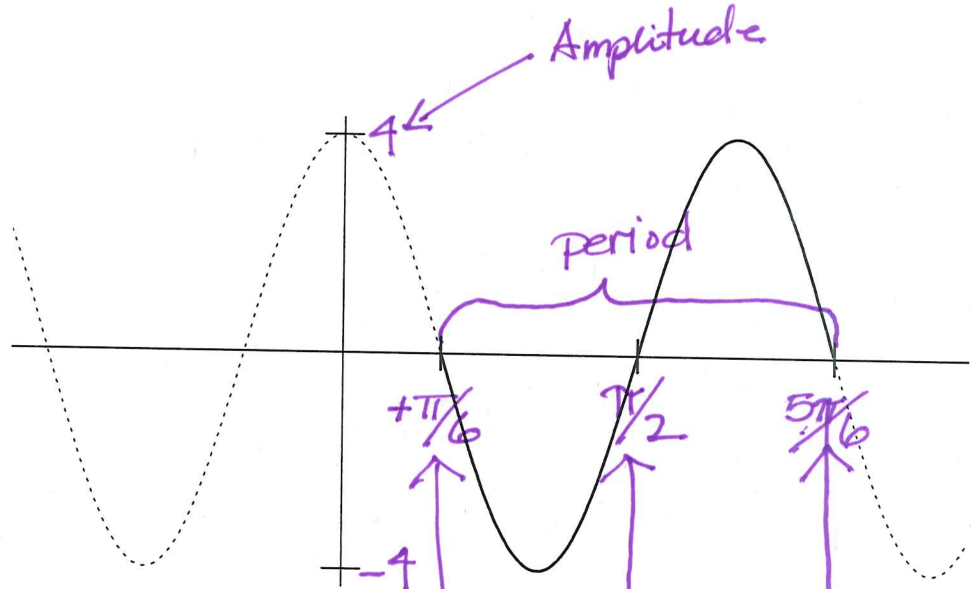
1. Find the period, amplitude, and shift for $f(x)$.

period $\Rightarrow \frac{2\pi}{b} = \frac{2\pi}{3}$

Amplitude $\Rightarrow |A| = |-4| = 4$

phase shift $\Rightarrow \frac{-c}{b} = \frac{+\pi/2}{3} = +\pi/6$

2. Label the five tick marks indicated on the $x-$ and $y-$ axes on the graph below.



phase shift

$\pi/6 + \frac{1}{2}$ period

$\pi/6 + \pi/3$

$\pi/6 + 2\pi/6$

$3\pi/6$

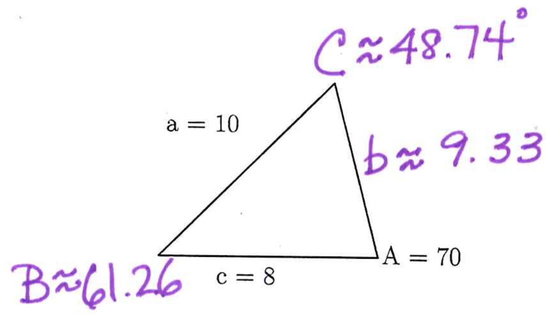
$\pi/2$

$+\pi/6 + 2\pi/3$

$\frac{\pi}{6} + \frac{4\pi}{6}$

$\frac{5\pi}{6}$

(13) Solve the triangle below. Make sure your calculator is in degree mode. (The dimensions given are not to scale.)



$A = 70°$

$a = 10$

$c = 8$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

$$\frac{10}{\sin(70°)} = \frac{8}{\sin(C)}$$

$$\sin(C) = \frac{8 \cdot \sin(70°)}{10}$$

$$C = \sin^{-1}\left[\frac{8\sin(70°)}{10}\right]$$

$$\boxed{C \approx 48.74}$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

$$\frac{10}{\sin(70°)} \approx \frac{b}{\sin(61.26°)}$$

$$\frac{10 \cdot \sin(61.26)}{\sin(70)} \approx b$$

$$9.33 \approx b$$

$B = 180 - A - C$

$B \approx 180 - 70 - 48.74$

$B \approx 61.26$

(14) Find the time it takes for an investment of \$1768 to double if it is put into an account with a 3.2% interest rate compounded continuously. Solve algebraically and show all work. Your answer should be exact.

$$A(t) = P_0 e^{rt}$$

$$A(t) = 1768 e^{.032t}$$

$$\frac{3536}{1768} = \frac{1768 e^{.032t}}{1768}$$

$$2 = e^{.032t}$$

$$\ln(2) = .032t$$

exact $\Longrightarrow$ $\boxed{\frac{\ln(2)}{.032}} = t$

$21.66\text{yrs.} \approx t$

approximate

(15) Given the function

$$f(x) = e^{-x}$$

(a) Find the average rate of change of $f(x)$ as x changes from $x = \overset{a}{-1}$ to $x = \overset{b}{2}$..

$$\text{A.R.O.C.} \Longrightarrow \frac{f(b)-f(a)}{b-a}$$

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{e^{-2}-e^{-(-1)}}{2+1} = \frac{\frac{1}{e^2} - \frac{e\cdot e^2}{1\cdot e^2}}{3} = \frac{\frac{1-e^3}{e^2}}{3}$$

$$\longrightarrow \frac{1-e^3}{e^2}\cdot\frac{1}{3} = \boxed{\frac{1-e^3}{3e^2}} \Longleftarrow \text{exact!}$$

(b) What does the number found in part (a) represent on the graph of $f(x)$?

The A.R.O.C. is the slope of the secant line joining the points $(-1, e)$ and $(2, e^{-2})$ on the graph of $f(x)$.

(16) (a) Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for

$$f(x) = 2x^2 - x + 5.$$

and simplify the results.

$$f(x+h) = 2(x+h)^2 - (x+h) + 5$$
$$= 2(x^2+2xh+h^2) - x - h + 5$$
$$= 2x^2 + 4xh + 2h^2 - x - h + 5$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(\cancel{2x^2} + 4xh + 2h^2 \cancel{-x} - h \cancel{+5}) - (\cancel{2x^2} \cancel{-x} \cancel{+5})}{h}$$

$$= \frac{4xh + 2h^2 - h}{h}$$

$$= \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}}$$

$$= 4x + 2h - 1$$

(17) Simplify $\sin(x + \pi)$ by using the sum ~~difference~~ of angles formula. Your answer should not contain π.

$$\sin(x+\pi) = \sin(x)\cos(\pi) + \cos(x)\sin(\pi)$$
$$= \sin(x)\cdot -1 + \cos(x)\cdot 0$$
$$= -\sin(x) + 0$$
$$= -\sin(x)$$

Free Response Questions: Show your work!

Some Useful Formulas

$$B(t) = P\left(1+r\right)^t$$

$$B(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$P(t) = P_0e^{rt}$$

$$Q(t) = Q_0e^{-rt}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$