3 The Cartesian Coordinate System Worksheet

Concepts:

- The Cartesian Coordinate System
- Graphs of Equations in Two Variables
- x-intercepts and y-intercepts
- Distance in Two Dimensions and the Pythagorean Theorem
- Equations of Circles
  - The Distance Formula and the Standard Form for an Equation of a Circle.
  - Writing Equations of Circles
  - Identifying Equations of Circles
- Midpoints
  - Finding Midpoints
  - Verifying that a Point Is the Midpoint of a Line Segment
- Steepness
- Rates of Change
- Lines
  - The Slope
  - The Slope as a Rate of Change
  - Linear Equations
  - Point-Slope Form
  - Vertical Lines
  - Horizontal Lines
  - Parallel Lines and Perpendicular Lines
- Using 2-Dimensional Graphs to Approximate Solutions of Equations in One Variable.
  - The Intersection Method
  - The Intercept Method

(Sections 1.3-1.4)

1. Is $(3, 2)$ on the graph of $x^2 - y^3 = 1$?

2. Is $(0, 1)$ on the graph of $x^2 - y^3 = 1$?

3. Is $(0, -1)$ on the graph of $x^2 - y^3 = 1$?

4. Find the intercepts of the graph of $x^2 - y^3 = 1$.

5. Find the point on the x-axis that is equidistant to $(2, 5)$ and $(-1, 3)$.

6. Find the point on the y-axis that is equidistant to $(2, 5)$ and $(-1, 3)$.

7. Find the perimeter of the triangle with vertices $A(-2, -5)$, $B(-2, 7)$, and $C(10, 10)$.

8. Find the area of the triangle with vertices $A(-2, -5)$, $B(-2, 7)$, and $C(10, 10)$.

9. Sketch the graph of the circle defined by $(x + 3)^2 + y^2 = 9$. What are the center and radius of this circle?

10. Is the graph of $x^2 + 6x + y^2 - 10y + 26 = 0$ a circle? If so, find its center and radius.

11. Is the graph of $4x^2 - 8x + 4y^2 + 4y - 23 = 0$ a circle? If so, find its center and radius.

12. Is the graph of $x^2 - 2x + y^2 + 8y + 26 = 0$ a circle? If so, find its center and radius.

13. Describe the graph of $x^2 + 4x + y^2 + 10y + 29 = 0$.

14. A diameter of a circle has endpoints $(1, -2)$ and $(3, 6)$. Find an equation for the circle.

15. The center of a circle is $(5, \frac{1}{4})$, and circle passes through the point $(-2, 3)$. Find an equation for the circle.

16. The center of a circle is $(4, -5)$ and the circle intersects the x-axis at 2 and 6. Find an equation for the circle.

17. For each point, determine if the point is inside, outside, or on the circle

$$(x + 5)^2 + (y - 3)^2 = 36.$$

   (a) $(4, 2)$
   (b) $(-5, 0)$
   (c) $(1, 2)$

18. Which of the following are equations for the line through the points $P(1,5)$ and $Q(2,-3)$?

(a) $y+3=-8(x-2)$

(b) $y=-8x-4$

(c) $y=-8(x-1)+5$

(d) $y+3=\frac{-1}{8}(x-2)$

(e) $y+3=\frac{1}{8}(x-2)$

(f) $y-5=\frac{-1}{8}(x-1)$

(g) $y-5=\frac{1}{8}(x-1)$

(h) $y-5=-8(x-1)$

(i) $y+5=-8(x+1)$

(j) $y-5=-8x-1$

(k) $y-5=\frac{-1}{8}x-1$

19. **TRUE or FALSE:** The line through the points $(0,-1)$ and $(-1,4)$ is perpendicular to the line through the points $(2,-8)$ and $(7,-7)$.

20. **TRUE or FALSE:** The line through the points $(-5,-7)$ and $(-8,-5)$ is parallel to the line through the points $(-7,0)$ and $(-10,2)$.

21. Find an equation for the line that is parallel to $y=\frac{5}{6}x+4$ and passes through the point (0,12).

22. Find an equation for the line that is parallel to $y=\frac{5}{6}x+7$ and contains the point (3,21).

23. Find an equation for the line that is perpendicular to $y=\frac{5}{6}x+4$ and contains the point (0,14).

24. How many intersection points could a circle and a line have?

25. How many intersection points could the graphs of two lines have?

26. Find the points of intersection between the graphs of $y=x$ and $y=x^2+5$. Sketch the graphs.

27. Find the points of intersection between the graphs of $y=2$ and $y=x^2-3x$. Sketch the graphs.

28. Find the points of intersection between the graphs of $3x+5y=1$ and $2x-10y=7$. Sketch the graphs.

29. This problem was taken from the Precalculus textbook by David H. Collingwood and K. David Prince. It is available at `http://www.math.washington.edu/~m120/`.

    Two planes flying opposite directions (North and South) pass each other 80 miles apart at the same altitude. The northbound plane is flying 200mph (miles per hour) and the southbound plane is flying 150 mph. How far apart are the planes in 20 minutes? When are the planes 300 miles apart?

30. This problem was taken from the Precalculus textbook by David H. Collingwood and K. David Prince. It is available at `http://www.math.washington.edu/~m120/`.

A spider is located at the position $(1, 2)$ in a coordinate system where units on each axis are feet. An ant is located at the position $(15, 0)$ in the same coordinate system. Assume the location of the spider after t minutes $s(t) = (1 + 2t, 2 + t)$ and the location of the ant after t minutes is $a(t) = (15 - 2t, 2t)$.

(a) Sketch a picture of the situation indicating the locations of the spider and ant at times $t = 0, 1, 2, 3, 4, 5$ minutes. Label the locations of the bugs in your picture using the notation $s(0), s(1), \ldots, s(5)$, $a(0), a(1), \ldots, a(5)$.

(b) When will the x-coordinate of the spider equal 5? When will the y-coordinate of the ant equal 5?

(c) Where is the spider located when its y-coordinate is 3?

(d) Where is each bug located when the y-coordinate of the spider is twice as large as the y-coordinate of the ant?

(e) How far apart are the bugs when their x-coordinates coincide? Draw a picture indicating the locations of each bug when their x-coordinates coincide.

(f) A sugar cube is located at the position $(9, 6)$. Explain why each bug will pass through the position of the sugar cube. Which bug reaches the sugar cube first?