

## Worksheet 22 - Polar Graphs (§10.6)

In Exercises 1 - 20, plot the graph of the polar equation by hand. Carefully label your graphs.

- |   |   |
|---|---|
| 1. Circle: $r = 6 \sin(\theta)$               | 2. Circle: $r = 2 \cos(\theta)$         |
| 3. Rose: $r = 2 \sin(2\theta)$                | 4. Rose: $r = 4 \cos(2\theta)$          |
| 5. Rose: $r = 5 \sin(3\theta)$                | 6. Rose: $r = \cos(5\theta)$            |
| 7. Rose: $r = \sin(4\theta)$                  | 8. Rose: $r = 3 \cos(4\theta)$          |
| 9. Cardioid: $r = 3 - 3 \cos(\theta)$         | 10. Cardioid: $r = 5 + 5 \sin(\theta)$  |
| 11. Cardioid: $r = 2 + 2 \cos(\theta)$        | 12. Cardioid: $r = 1 - \sin(\theta)$    |
| 13. Limaçon: $r = 1 - 2 \cos(\theta)$         | 14. Limaçon: $r = 1 - 2 \sin(\theta)$   |
| 15. Limaçon: $r = 2\sqrt{3} + 4 \cos(\theta)$ | 16. Limaçon: $r = 3 - 5 \cos(\theta)$   |
| 17. Limaçon: $r = 3 - 5 \sin(\theta)$         | 18. Limaçon: $r = 2 + 7 \sin(\theta)$   |
| 19. Lemniscate: $r^2 = \sin(2\theta)$         | 20. Lemniscate: $r^2 = 4 \cos(2\theta)$ |

In Exercises 21 - 30, **compute** the exact polar coordinates of the points of intersection of graphs of the polar equations. Remember to check for intersection at the pole (origin).

- |   |   |
|---|---|
| 21. $r = 3 \cos(\theta)$ and $r = 1 + \cos(\theta)$       | 22. $r = 1 + \sin(\theta)$ and $r = 1 - \cos(\theta)$ |
| 23. $r = 1 - 2 \sin(\theta)$ and $r = 2$                  | 24. $r = 1 - 2 \cos(\theta)$ and $r = 1$              |
| 25. $r = 2 \cos(\theta)$ and $r = 2\sqrt{3} \sin(\theta)$ | 26. $r = 3 \cos(\theta)$ and $r = \sin(\theta)$       |
| 27. $r^2 = 4 \cos(2\theta)$ and $r = \sqrt{2}$            | 28. $r^2 = 2 \sin(2\theta)$ and $r = 1$               |
| 29. $r = 4 \cos(2\theta)$ and $r = 2$                     | 30. $r = 2 \sin(2\theta)$ and $r = 1$                 |

In Exercises 31 - 40, sketch the region in the  $xy$ -plane described by the given set.

- |  |  |
|--|--|
| 31. $\{(r, \theta) \mid 0 \leq r \leq 3, 0 \leq \theta \leq 2\pi\}$                                    | 32. $\{(r, \theta) \mid 0 \leq r \leq 4 \sin(\theta), 0 \leq \theta \leq \pi\}$            |
| 33. $\{(r, \theta) \mid 0 \leq r \leq 3 \cos(\theta), -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\}$ | 34. $\{(r, \theta) \mid 0 \leq r \leq 2 \sin(2\theta), 0 \leq \theta \leq \frac{\pi}{2}\}$ |

35.  $\{(r, \theta) \mid 0 \leq r \leq 4 \cos(2\theta), -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}\}$       36.  $\{(r, \theta) \mid 1 \leq r \leq 1 - 2 \cos(\theta), \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}\}$
37.  $\{(r, \theta) \mid 1 + \cos(\theta) \leq r \leq 3 \cos(\theta), -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}\}$
38.  $\{(r, \theta) \mid 1 \leq r \leq \sqrt{2 \sin(2\theta)}, \frac{13\pi}{12} \leq \theta \leq \frac{17\pi}{12}\}$
39.  $\{(r, \theta) \mid 0 \leq r \leq 2\sqrt{3} \sin(\theta), 0 \leq \theta \leq \frac{\pi}{6}\} \cup \{(r, \theta) \mid 0 \leq r \leq 2 \cos(\theta), \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}\}$
40.  $\{(r, \theta) \mid 0 \leq r \leq 2 \sin(2\theta), 0 \leq \theta \leq \frac{\pi}{12}\} \cup \{(r, \theta) \mid 0 \leq r \leq 1, \frac{\pi}{12} \leq \theta \leq \frac{\pi}{4}\}$

In Exercises 41 - 50, use set-builder notation to describe the polar region. Assume that the region contains its bounding curves.

41. The region inside the circle  $r = 5$ .
42. The region inside the circle  $r = 5$  which lies in Quadrant III.
43. The region inside the left half of the circle  $r = 6 \sin(\theta)$ .
44. The region inside the circle  $r = 4 \cos(\theta)$  which lies in Quadrant IV.
45. The region inside the top half of the cardioid  $r = 3 - 3 \cos(\theta)$
46. The region inside the cardioid  $r = 2 - 2 \sin(\theta)$  which lies in Quadrants I and IV.
47. The inside of the petal of the rose  $r = 3 \cos(4\theta)$  which lies on the positive  $x$ -axis
48. The region inside the circle  $r = 5$  but outside the circle  $r = 3$ .
49. The region which lies inside of the circle  $r = 3 \cos(\theta)$  but outside of the circle  $r = \sin(\theta)$
50. The region in Quadrant I which lies inside both the circle  $r = 3$  as well as the rose  $r = 6 \sin(2\theta)$

There are some important polar curves which are simply too difficult to graph by hand and that makes the calculator an important tool for your further studies in Mathematics, Science and Engineering. Below is a brief demo of how to use the graphing calculator to plot polar curves. The first thing you must do is switch the MODE of your calculator to POL, which stands for “polar”.

