Calculus I MA 113
Special exam 1 May 2006

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).* 3) exact answers are preferred.
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name _____

School _____

Question	Score	Total
Q1		12
Q2		12
Q3		12
Q4		12
Q5		12
Q6		12
Q7		12
Q8		12
Free	4	4
		100

1. For each of the following functions, compute the derivative and simplify.

   (a) $f(x) = \dfrac{x}{2 + x^2}$

   (b) $g(x) = \sqrt{(x^2 + 4x)}$

   (c) $h(x) = \cos(\pi x^2)$

2. (a) State the intermediate value theorem.

   (b) Find an interval $[a, b]$ so that the equation $x^3 + 3x^2 = \cos(x)$ has a solution on the interval $[a, b]$. Use the intermediate value theorem to show that your solution is correct.

3. Suppose that x and y are two numbers whose sum is 12. Find the maximum value of x^3y.

4. Evaluate the following definite integrals.

(a) $\displaystyle\int_{-2}^{2} (x^3 + x^2 + x)\, dx$

(b) $\displaystyle\int_{0}^{3} \frac{x}{\sqrt{x^2 + 16}}\, dx$

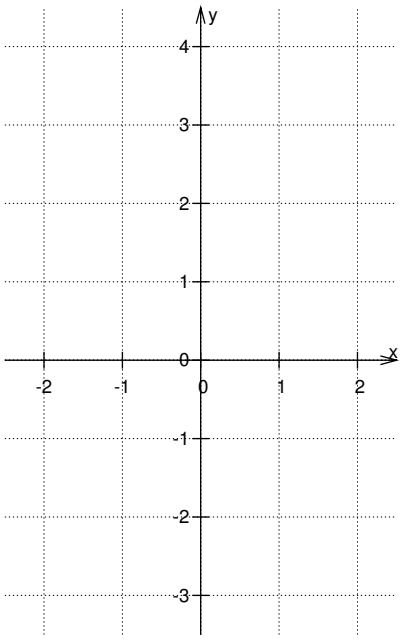
(c) $\displaystyle\int_{-1}^{3} |x + 2|\, dx$

5. Consider the functions $f(x) = 2x + 1$ and $g(x) = x^3 + x + 1$.

   (a) Find the coordinates (x, y) of the points where the graphs of f and g intersect.

   (b) Make a rough sketch of the graphs of f and g.

   (c) Find the area enclosed by the graphs of the functions f and g.

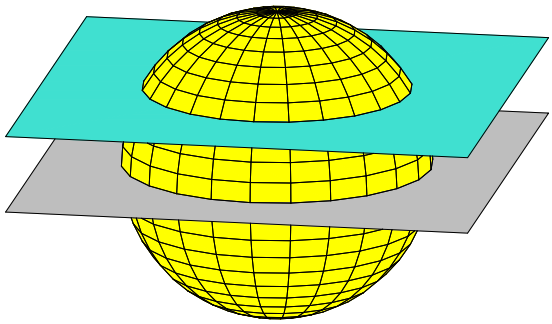


6. A sphere of radius 4 is cut by two parallel planes. One plane passes through the center of the sphere and the planes are 2 units apart.

   (a) For each plane, give the area of the circle which is the intersection of the plane and the sphere.

   (b) Write an integral that gives the volume of that part of the sphere that lies between the two planes.

   (c) Find the volume of the part of the sphere that lies between the two planes.



7. Consider the function $f(x) = (2 - x^2)^2$.

   (a) Find the x-intercepts and y-intercepts for the graph of f.

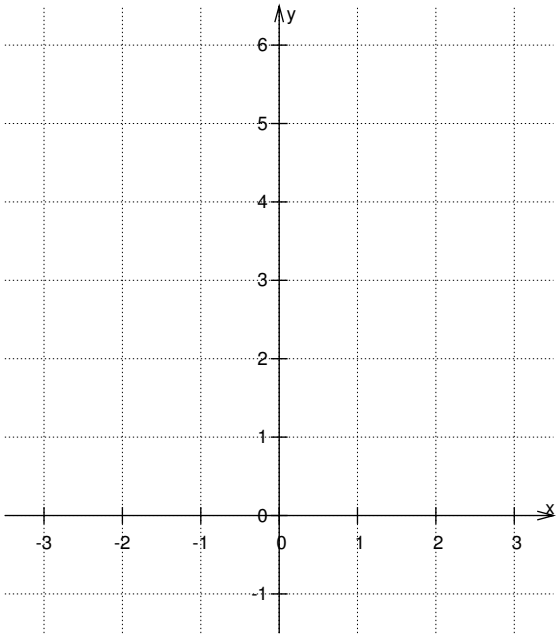
   (b) Find the derivatives $f'(x)$ and $f''(x)$.

   (c) Find the intervals on which f is increasing and decreasing.

   (d) Find the locations of the local maxima and minima for f.

   (e) Find the intervals on which f is concave up and concave down.

   (f) Sketch a graph for f which reflects the information



8. Consider the integral $\int_0^2 (1+x^2)\,dx$.

   (a) Compute the Riemann sum for this integral obtained when we divide the interval $[0, 2]$ into 4 equal subintervals. Use the right endpoint of each subinterval as the sample points.

   (b) On the graph below sketch the rectangles whose area we find when we compute the Riemann sum.

   (c) Is the value of the Riemann sum smaller or larger than the value of the integral?