Calculus I
Bypass examination

UK Department of Mathematics
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Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may not receive credit).** 3) give exact answers, rather than decimal approximations to the answer.
Please write out the work you used to find the solution and then put the answer in the space provided.

Name ______________________________
School ______________________________
Calculus teacher ______________________________

Question	Score	Total
1		6
2		6
3		6
4		6
5		10
6		6
7		8
8		10
9		10
10		10
11		12
12		10
		100

1. Find the derivatives of the following functions.

   (a) $f(x) = \dfrac{1}{\sqrt{x^2+1}}$.

   (b) $g(x) = \sin(x^2)$.

(a) __________________________, (b)__________________________

2. Find the equation of the tangent line to the graph of $y = x^2 + 3x$ at the point $(-1, -2)$. Put your answer in the form $y = mx + b$.

$y =$__________________________

3. Suppose f and g are continuous functions whose domain is the interval $(0, 8)$ and we have

$$\lim_{x\to 4} f(x) = 3, \quad \lim_{x\to 5} f(x) = 7, \quad \lim_{x\to 4} g(x) = 5, \quad \text{and} \quad \lim_{x\to 5} g(x) = 2.$$

Find the limits.

(a) $\lim_{x\to 4}(f(x) + g(x))$
(b) $\lim_{x\to 5}(xf(x) + g(x))$
(c) $\lim_{x\to 4} f(g(x) - 1)$

(a) ________________, (b) ________________ (c) ________________

4. Let

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 2}, & x \neq 2 \\ A, & x = 2. \end{cases}$$

Find the value of A for which f will be continuous.

$A =$________________

5. (a) Give the definition of the derivative of a function f at a point a.

(b) Use the definition to find $f'(3)$ for the function f defined by

$$f(x) = \frac{x}{x+1}.$$

Answer________________________

6. The area of a circle is increasing at a rate of 3 meters^2/second. Find the rate of change of its radius with respect to time when the radius is 2 meters.

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7. Let $f(x) = x^3 - 6x^2 + 20$.

   (a) Find the critical numbers for f.

   (b) Find the locations of local maxima and minima for f. Explain your answer.

(a) Critical numbers at $x =$____________________

(b) Local maxima at $x =$____________________

Local minima at $x =$____________________

8. A particle moves along the x-axis so that at time t seconds it is $x(t) = -t^3 + 3t^2$ meters to the right of the origin.

   (a) Find the position, velocity and acceleration of the particle at time $t = 2$ seconds.

   (b) Is the particle moving to the right or the left at time $t = 4$ seconds?

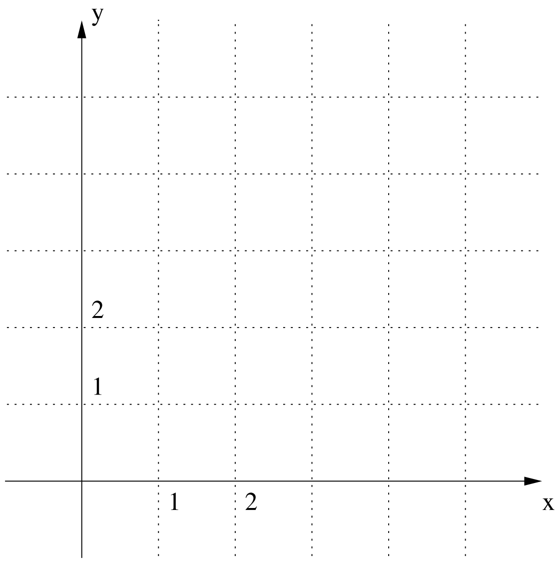
   (c) What is the farthest that the particle is from the origin for t in the interval $[0, 3]$?

(a) Position ____________________, Velocity ____________________,

Acceleration ____________________

(b) __________________________ (c) __________________________

9. Sketch the graphs of the curves defined by $y = x + 1$ and $y = (x - 1)^2$. Find the area between these curves.



The area is ____________________

10. Evaluate the following definite integrals. Be sure to show all your work.

$$(a) \quad \int_0^1 t\sqrt{1 + 4t^2}\, dt \qquad (b) \int_0^{\pi/2} \cos(t)\ \sin^2(t)\, dt$$

(a) ____________________ , (b) ____________________

11. A company wishes to manufacturer a box with a square base and a volume of 625 cubic centimeters. The material for the top and bottom costs \$0.10 per square centimeter and the material for the sides costs \$0.02 per square centimeter. Find the dimensions and cost of the cheapest such box. Explain how you know you have found the cheapest box.

Dimensions________________________

Cost________________________

12. Let

$$F(x) = \int_1^x \frac{1}{2+\sin(t)}\,dt.$$

(a) Find the derivative $F'(x)$.

(b) Find the intervals where the function F is concave up or concave down.

(a) $F'(x) =$ ______________________

(b) Intervals where F is concave up ______________________

Intervals where F is concave down ______________________