Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*

Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

You are to answer two of the last three pages. Please indicate which page is not to be graded by drawing a line through its number on the table below. If you do not cross out one of the pages, we may assign credit for the two pages with the lowest scores.

Name Key.

Section ____________

Last four digits of student identification number ____________

	Score	Possible
Page 1	14	14
Page 2	14	14
Page 3	14	14
Page 4	14	14
Page 5	13	14
Page 6/Question 8	14	14
Page 7/Question 9	14	14
Page 8/Question 10	14	14
Free	2	2
	99	100

No one is perfect.

1. Find the equation of a line passing through the point $(1, 7)$ which is perpendicular to the line $6x + 3y = 9$. Express your answer in the form $y = mx + b$.

Slope of line $6x+3y=9$

$3y = 9-6x$

$y = 3-2x$

② slope is -2

② Perpendicular slope is $+1/2$

line through $(1,7)$ with slope $+1/2$ is

② $y - 7 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x - \frac{1}{2} + 7$

① $= \frac{1}{2}x + \frac{13}{2}$

Answer: $y = \frac{1}{2}x + 13/2$

2. Let f and g be functions which are defined by $f(x) = \sqrt{x-1}$ and $g(x) = 1/x$.

(a) Find the composition of g and f, $g \circ f$.

(b) Give the domain of the composite function $g \circ f$.

(a) $g \circ f(x) = 1/\sqrt{x-1}$.

(b) Domain of $g \circ f$ is all x s.t. $x-1>0$ or $x>1$. $\{x: x>1\}$ or $(1,\infty)$.

④

(a) $g \circ f(x) =$ $1/\sqrt{x-1}$

③ (b) The domain of $g \circ f$ is $\{x: x>1\}$. or $(1,\infty)$

Deduct ① for improper set notation.

3. At time $t = 0$ seconds, a diver jumps from a diving board that is 10 meters above the water. The height of the diver measured in meters above the water at time t seconds is given by the function $s(t) = -5t^2 + 5t + 10$. Find the average velocity of the diver between $t = 1$ second and $t = 3/2$ seconds.

Average velocity $= \dfrac{s(3/2) - s(1)}{3/2 - 1}$ (2)

$s(3/2) = -5 \cdot \frac{9}{4} + \frac{15}{2} + 10 = -\frac{45}{4} + \frac{30}{4} + \frac{40}{4} = 25/4$ (1)

$s(1) = -5 + 5 + 10. = 10.$ (1)

$\dfrac{s(3/2) - s(1)}{3/2 - 1} = \dfrac{25/4 - 10}{1/2} = 2 \cdot \left(\frac{-15}{4}\right). = -15/2$ (2)

Average velocity = $-15/2$ meter/second. (1) units.

4. Let P_1, P_2, P_3, ... be a sequence of statements. Suppose it is known that:

- The statement P_3 is true.
- If P_n is true, then P_{n+2} is true.

Which of the statements P_2, P_3, P_4, P_5, P_6, P_7 and P_8 must be true? If none of these statements must be true, write "none" as the answer.

(2) P_3 is true

(2) If P_3 is true, then P_5 is true

(2) If P_5 is true, then P_7 is true.

(1) explanation.

Deduct (2) point for each extra answer.

Answer: P_3, P_5, P_7.

5. Use our rules and theorems for limits to find the following limits. Be sure to give a clear statement of your reasoning.

(a) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$

(b) $\lim_{x\to 2} \dfrac{x^3-27}{x+2}$

(c) $\lim_{x\to 7} \dfrac{x^2-49}{x-7}$

(a) $\dfrac{\sqrt{x}-1}{x-1} = \dfrac{(\cancel{\sqrt{x}-1})}{(\cancel{\sqrt{x}-1})(\sqrt{x}+1)}$. As $\lim_{x\to 1} \sqrt{x}+1 = 2$

③ simplify
② Value
① Reason

the quotient rule for limits tells us $\lim_{x\to 1} \dfrac{1}{\sqrt{x}+1} = 1/2.$

(b) $\lim_{x\to 2} \dfrac{x^3-27}{x+2} = \dfrac{8-27}{4} = -19/4$ Since the denominator has non-zero limit, the result follows by rule for limit of quotient.

③ Value
① Reason

—or— Direct substitution property for rational funct

(c) $\lim_{x\to 7} \dfrac{x^2-49}{x-7} = \lim_{x\to 7} \dfrac{(x+7)\cancel{(x-7)}}{\cancel{(x-7)}} = 14.$ ③ ②

③ simplify
② Answer

(a) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1} =$ 1/2, (b) $\lim_{x\to 2} \dfrac{x^3-27}{x+2} =$ -19/4,

(c) $\lim_{x\to 7} \dfrac{x^2-49}{x-7} =$ 14.

6. (a) Using the definition of the derivative, find the derivative of the function $f(x) = x^2 + 2$ at $x = 1$.

(b) Find the equation of the tangent line to the graph of the function $f(x) = x^2 + 2$ at the point $(1, 3)$. Put the equation of the tangent line in the form $y = mx + b$.

a) Derivative at $x=1$ is

$$\lim_{h \to 0} \frac{(1+h)^2 + 2 - (1+2)}{h} \quad (3)$$

$$\frac{(1+h)^2 + 2 - 3}{h} = \frac{1 + 2h + h^2 + 2 - 3}{h} = \frac{2h + h^2}{h} = 2 + h$$

$$\lim_{h \to 0} (2+h) = \lim_{h \to 0} 2 + \lim_{h \to 0} h = \underline{2.} \quad (2)$$

$\underline{f'(1) = 2}$

b) Tangent line: has slope $f'(1) = 2$ (2)
and passes through $(1, f(1)) = (1, 3)$ (2).
Equation is $y - 3 = 2(x - 1)$ (2)
$y = 2x - 2 + 3 = 2x + 1$. (1)

(a) $f'(1) =$ 2

(b) $y = 2x + 1.$

7. Below, find the graph of a function f on the interval $[0, 4]$.

(4) 1 pt. each answer

(a) Use the graph of f to give the value of the limits.

$\lim_{x \to 1^-} f(x) =$ 0, $\lim_{x \to 1^+} f(x) =$ 1

$\lim_{x \to \frac{3}{2}^-} f(x) =$ 1, $\lim_{x \to \frac{3}{2}^+} f(x) =$ 1

(b) Is f continuous at $x = 1$? Explain.

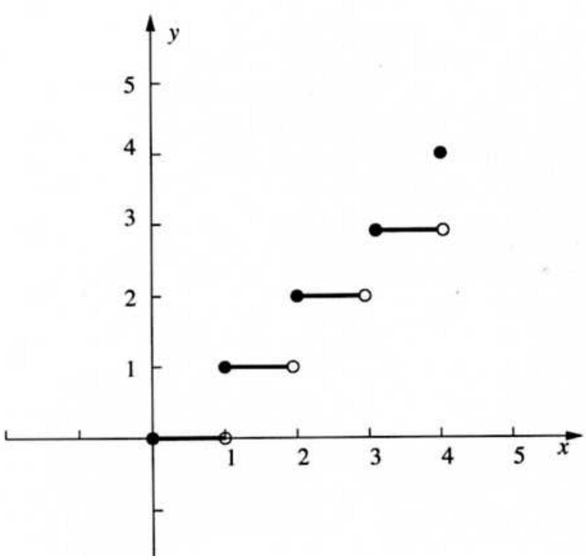
(3) No. $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$, hence $\lim_{x \to 1} f(x)$ does not exist.

Reason (2)

(c) Is f continuous at $x = 3/2$? Explain.

(3) Yes, ~~for~~ as $\lim_{x \to \frac{3}{2}^+} f(x) = \lim_{x \to 3/2^-} f(x) = 1$, we have that $\lim_{x \to 1} f(x) = 1$.
Since $f(3/2) = 1$, ~~also~~ f is continuous.

Reason (2)



Answer two of the following three questions. Indicate clearly which question is not to be graded by drawing a line through the question number in the table on the front of the exam.

8. (a) Give the definition of "continuity of a function f at a number a". Use complete sentences.

   (b) Let

$$g(x) = \begin{cases} x^2, & x > 3 \\ cx + 2, & x \leq 3 \end{cases}$$

   where c is a number.

   Find $\lim_{x \to 3+} g(x)$ and $\lim_{x \to 3-} g(x)$. The value for one of these limits will depend on the unknown number c.

   (c) Find the value of c so that the function g is continuous at 3. Explain why g is continuous for this value of c.

(a) A function f is continuous at a if $\lim_{x \to a} f(x) = f(a)$. 4.

Deduct 1 if answer is not a sentence.

(b) $\lim_{x \to 3^+} g(x) = 9$. 2

2

$\lim_{x \to 3^-} g(x) = 3c + 2$.

3

(c) $9 = 3c + 2$

$7 = 3c$

3 $7/3 = c$.

If $9 = 3c + 2$, then $\lim_{x \to 3^+} g(x) = \lim_{x \to 3^-} g(x) = 9$.

Also, $g(3) = 3c + 2 = 9$.

Thus $\lim_{x \to 3} g(x) = g(3) = 9$.

9. (a) Give the definition of the derivative $f'(x)$ of a function f at a point x. Use complete sentences.

(b) Use the definition to find the derivative, $g'(x)$, of the function $g(x) = \dfrac{1}{5x+2}$.

(c) Give the domain of $g'(x)$.

1(a) The derivative of a function f at a number x is

4

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$ provided the limit exists.

Deduct 1 not a sentence

(b) Simplify difference quotient:

3

$$\frac{g(x+h) - g(x)}{h} = \frac{1}{h}\left(\frac{1}{5(x+h)+2} - \frac{1}{5x+2} \right)$$

$$= \frac{1}{h}\left(\frac{5x+2 - (5x+5h+2)}{(5x+5h+2)(5x+2)} \right).$$

$$= \frac{1}{h} \frac{-5h}{(5x+2)(5x+5h+2)}.$$

$$= \frac{-5}{(5x+2)(5x+5h+2)}.$$ 3

$$\lim_{h \to 0} \frac{-5}{(5x+2)(5x+5h+2)} = \frac{-5}{(5x+2)^2},$$ by the quotient rule for limits if $5x+2 \neq 0$. 1 Reason

(c) Domain of $g'(x)$ is $\{x : x \neq -2/5\}$ or $(-\infty, -2/5) \cup (-2/5, \infty)$. 3

10. (a) State the intermediate value theorem. Use complete sentences.

(b) Find a closed interval $[a, b]$ so that the equation $x^5 - 2x^3 - 4 = x$ has a solution in the open interval (a, b). Use the intermediate value to explain why you know there is a solution to the equation in the interval you found.

2

a) Suppose f is a function which is continuous on the closed interval $[a,b]$. Let M be a number between $f(a)$ and $f(b)$. 2 Then there is a number c in (a,b) so that $f(c)=M$. 1

b) Let $f(x) = x^5-2x^3-x-4$. 1

As f is a polynomial, f is continuous 2 everywhere.

$f(0) = -4$. 2

$f(2) = 32-16-2-4 = 10$. 2

As $f(0)<0$ and $f(2)>0$, there is a number c in the interval 2 $(0,2)$ so that $f(c)=0$.