MA 113 — Calculus I Fall 2009
Exam 1 September 22, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ________________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		13
2		10
3		8
4		8
5		8
6		9
7		9
8		16
9		16
10		16
Free	3	3
		100

(1) Calculate the following limits or show that they do not exist. If the limit in question does not exist, but is ∞ or $-\infty$, then clearly indicate that.

(a) $\displaystyle\lim_{x\to 4} \frac{x^2-5x+4}{x^2-3x-4}$

(b) $\displaystyle\lim_{x\to 4} \frac{x^2+5x+4}{x^2+3x-4}$

(c) $\displaystyle\lim_{x\to 3} \frac{5-x}{(x-3)^2}$.

(d) $\displaystyle\lim_{h\to 0} \frac{(h-1)^2-1}{h}$.

(a) ______________________________

(b) ______________________________

(c) ______________________________

(d) ______________________________

(2) (a) Solve the equation $2^{x-7} = 3$. Give the **exact** answer.

(b) Express the quantity

$$\log_3(1 + x^2) + \frac{1}{2}\log_3(x) - \log_3(4x)$$

as a single logarithm.

(a) ______________________________

(b) ______________________________

(3) Consider the functions $f(x) = \sqrt{10 - x}$ and $g(x) = x^2+1$. Let h be the composite function $h(x) = (f \circ g)(x)$.

(a) Compute $h(2)$.

(b) Find the domain of h. **As usual, justify your answer.**

(a) $h(2) =$ ____________________

(b) Domain of h is ____________________

(4) Let f be a function such that, for all real numbers x near 5,

$$\frac{1}{5}x + \frac{5}{x} + 2 \leq f(x) \leq x^2 - 10x + 29.$$

Argue that $\lim_{x \to 5} f(x)$ exists and find its value. **As usual, justify your answer.**

$\lim_{x \to 5} f(x) =$ ______________________________

(5) Let f and g be two functions such that the following limits exist

$$\lim_{x\to 3} g(x) = 6, \quad \lim_{x\to 3} \left[xf(x) - 2^x g(x)\right] = 12.$$

Use the limit laws to compute the following limits.

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-4}{g(x)}$.

(b) $\displaystyle\lim_{x\to 3} f(x)$.

(a) ______________________________

(b) ______________________________

(6) Consider the function

$$f(x) = \frac{2x-1}{4x-6}.$$

(a) Find the domain of f.

(b) Find the inverse function f^{-1} of f.

(a) Domain of f is ________________________

(b) $f^{-1}(x) =$ ____________________________.

(7) An object is moving on a straight line so that at time t seconds it is located at

$$s(t) = t^2 + 5t$$

meters to the right of some reference point. In the following problems also give the units with your answers.

(a) Find the average velocity of the object for the time interval $2 \le t \le 4$.

(b) Find the average velocity for the time interval $[2, t]$, where $t > 2$. **Simplify your answer.**

(c) Use your answer in (b) to find the instantaneous velocity of the object at time $t = 2$.

(a) Average velocity over $[2, 4]$ is ____________________

(b) Average velocity over $[2, t]$ is ____________________

(c) Instantaneous velocity at time $t = 2$ is _______________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) Define what it means for a function f to be continuous at a. Use complete sentences.

Let

$$f(x) = \begin{cases} cx^2 - 2x + 9, & \text{if } x < 2, \\ 8, & \text{if } x = 2, \\ \frac{4}{x} + 3c, & \text{if } x > 2. \end{cases}$$

For the following problems, always justify your answer!

(b) Find all values for c such that $\lim_{x \to 2} f(x)$ exists.

(c) For which of the values for c found in (b) is the function f continuous at 2?

(d) Find all values for c such that the function f is continuous at 0.

(b) ______________ (c) ______________ (d) ______________

(9) (a) State the Intermediate Value Theorem. Use complete sentences.

(b) **Explain in detail why and how** you can use this theorem to show that the equation

$$x^4 - 5x^2 = 2$$

has a solution in the interval (2, 3).

(10) (a) State the definition of the derivative of a function at a point a. Use complete sentences.

Consider now the function

$$f(x) = \sqrt{x+8}.$$

(b) Compute the slope of the secant line through the points $(1, f(1))$ and $(8, f(8))$.

(c) Compute the slope of the secant line through the points $(1, f(1))$ and $(1+h, f(1+h))$.

(d) Use part (c) to compute $f'(1)$.

(e) Compute the equation of the tangent line to the graph of f at the point $(1, f(1))$. Put your answer in the form $y = mx + b$.