MA 113 Calculus I Fall 2013
Exam 1 Tuesday, 24 September 2013

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

[B,D,E,C,C, C,E,A,D,E]

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the line that passes through the point $(-2, 2)$ and is perpendicular to the line $2x + y = 1$.

   (A) $y = -2x - 2$

   (B) $y = \frac{1}{2}x + 3$

   (C) $y = 2x + 6$

   (D) $y = -\frac{1}{2}x + 1$

   (E) $y = -2x + 1$

2. Suppose that f is a function of the form $f(x) = Ae^{kx}$, $f(1) = 3$ and $f(3) = 12$. Find A and k.

   (A) $A = 3/2$, $k = 2$

   (B) $A = 3$, $k = 2$

   (C) $A = 6$, $k = 2$

   (D) $A = 3/2$, $k = \ln(2)$

   (E) $A = 3$, $k = \ln(2)$

3. Suppose that $\cos(\theta) = x$ and $\pi < \theta < 2\pi$. Find $\cos(-\theta)$ and $\sin(-\theta)$.

   (A) $\cos(-\theta) = \sqrt{1 - x^2}$, $\sin(-\theta) = x$

   (B) $\cos(-\theta) = -x$, $\sin(-\theta) = -\sqrt{1 - x^2}$

   (C) $\cos(-\theta) = x$, $\sin(-\theta) = -\sqrt{1 - x^2}$

   (D) $\cos(-\theta) = -x$, $\sin(-\theta) = \sqrt{1 - x^2}$

   (E) $\cos(-\theta) = x$, $\sin(-\theta) = \sqrt{1 - x^2}$

Record the correct answer to the following problems on the front page of this exam.

4. If $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3}(f(x)g(x)) = -2$ find

$$\lim_{x\to 3}(2f(x) + g(x)).$$

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} 2x-1, & x < 2 \\ c, & x = 2 \\ x, & x > 2 \end{cases}$$

and we know that f is left-continuous at 2. Find c.

(A) 1

(B) 2

(C) 3

(D) 4

(E) There is not enough information to answer this question.

Record the correct answer to the following problems on the front page of this exam.

6. Let the position of a particle be given by $p(t) = t^2$. Find the time a so that the average velocity of the particle for $2 \le t \le a$ is equal to 6.

   (A) 2

   (B) 3

   (C) 4

   (D) 5

   (E) There is more than one correct answer.

7. Use the graph of f below to determine which of the following statements is true.

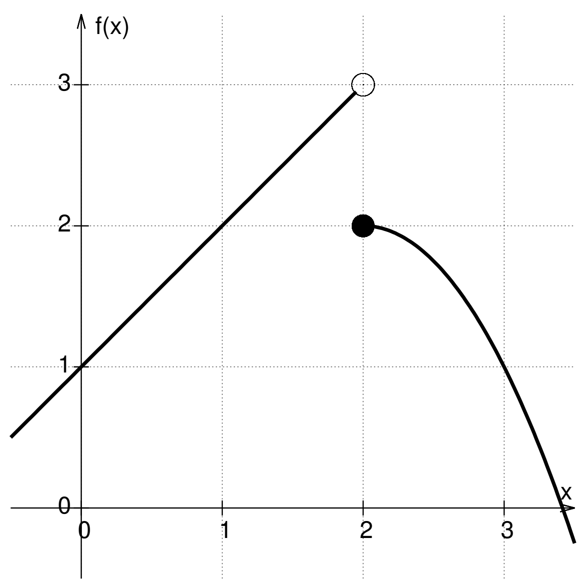
   (A) f is continuous at $x = 2$

   (B) $\lim_{x\to 2} f(x) = 2$

   (C) $\lim_{x\to 2^+} f(x) = 3$

   (D) $\lim_{x\to 2^-} f(x) = f(2)$

   (E) $\lim_{x\to 2^-} f(x) = 3$



Record the correct answer to the following problems on the front page of this exam.

8. Let f and g be functions that are defined on the real line. Four of the statements are true for any functions f and g. One of the statements may fail for some choices of f and g. Which of the statements may be false?

   (A) If $f(1) = g(1)$, then $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$.

   (B) If $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$, then $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} g(x)$.

   (C) If $\lim_{x\to 1} f(x) = 7$ and f is continuous at $x = 1$, then $f(1) = 7$.

   (D) If $\lim_{x\to 1} f(x) = 2$ and $\lim_{x\to 1} g(x) = 3$, then $\lim_{x\to 1}(3f(x) - 2g(x)) = 0$.

   (E) If $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^-} g(x) = 2$, then $\lim_{x\to 1} g(x) = 2$.

9. Suppose that $f(x) = (x-1)^2$, $h(x) = 0$, g is a function with domain $(-\infty, \infty)$, and $h(x) \leq g(x) \leq f(x)$ for all x. Then we can use the squeeze theorem to find

   (A) $\lim_{x\to 0} g(x) = 0$

   (B) $\lim_{x\to 0} g(x) = -1$

   (C) $\lim_{x\to 0} g(x) = 1$

   (D) $\lim_{x\to 1} g(x) = 0$

   (E) $\lim_{x\to 1} g(x) = 1$

10. Find the value of the limit $\displaystyle\lim_{x\to 0} \frac{\tan(2x)}{x}$.

   (A) -2

   (B) -1

   (C) 0

   (D) 1

   (E) 2

Free Response Questions: Show your work!

11. A rectangle has perimeter of 30 meters.

    (a) Make a sketch of the rectangle and label the sides in your sketch. Write a function A that gives the area of the rectangle as a function of the length of one of the sides.

    (b) Give the domain of the function A. The sides of a rectangle cannot be of negative length, but we allow the cases where one of the sides is of length 0.

Let x, y be sides of the rectangle. Perimeter condition gives $2x + 2y = 30$ or $y = 15 - x$. Area equals $xy = x(15 - x)$.	Total 8pts. Sketch 2 pts., Perimeter formula and solving for one variable $2 + 2$ pts. , Final area formula 2 pts.
Domain is $\{x \mid 0 \leq x \leq 15\}$ or $[0, 15]$.	One point for each correct endpoint.

Free Response Questions: Show your work!

12. Let $f(x) = \dfrac{x+2}{x-3}$. Find the inverse function f^{-1} and the domain and range of f^{-1}.

Start $y = \dfrac{x+2}{x-3}$ or $x = \dfrac{y+2}{y-3}$ optional. Manipulate to solution $x = \dfrac{3y+2}{y-1}$ or corresponding expression with x, y swapped. Report answer: $f^{-1}(x) = \dfrac{3x+2}{x-1}$. No penalty for consistent swap of x to y. Domain $(-\infty, 1) \cup (1, \infty)$ and Range $(-\infty, 3) \cup (3, \infty)$.	Start 2 pts. Manipulate 4 pts. One or two points off for each mistake. Report 2 pts. Domain/Range 1+1 pts.

Free Response Questions: Show your work!

13. Let $f(x) = \dfrac{1}{2x-1}$.

    (a) Find the slope of the line that passes through the points $(1, f(1))$ and $(1+h, f(1+h))$. Simplify your answer.

    (b) Take the limit as h tends to zero of the expression found in part a). Use the limit laws to justify your evaluation of the limit.

    (c) Give the geometric interpretation of your answer to part (b).

Expression $\dfrac{\frac{1}{2(1+h)-1} - \frac{1}{2(1)-1}}{(1+h)-1}$. Algebraic simplification: Num. $\dfrac{-2h}{2h+1}$. Den. h. Slope simplified: $\dfrac{-2h}{(2h+1)(h)} = \dfrac{-2}{2h+1}$.	Total 6 pts. Expression 2 pts. Simplification and slope 4 pts.
Limit $\lim_{h\to 0} \dfrac{-2}{2h+1} = \dfrac{-2}{1} = -2$, using quotient rule applied to polynomials. (Or rational function rule.)	2 pts.
It gives the slope of tangent at $x = 1$ or at the point $(1, f(1)) = (1, 1)$.	2 pts.

Free Response Questions: Show your work!

14. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

(a) $\lim_{x \to 1} \dfrac{e^x(x-1)}{x^2-1}$.

Optional: Observe $\frac{0}{0}$ form. Simplify algebraically to $\dfrac{e^x}{x+1}$. Limit by quotient rule or direct substitution rule $e/2$.	Simplify 1 pt, explanation 2 points, limit 2 pts.

(b) $\lim_{x \to 1} \dfrac{e^x(x+1)}{x^2-1}$.

Observe that the limit of denominator is zero. Simplify algebraically to $\dfrac{e^x}{x-1}$. (Simplification is not required, but encouraged.) For the left limit $\lim_{x \to 1^-}$ we note numerator stays positive and denominator goes to zero through negative values. So, the limit is $-\infty$. For the right limit $\lim_{x \to 1^+}$ it reverses to $+\infty$. Since the one sided limits are ∞ and $-\infty$, the limit does not exist.	Explanation 3 points.
Alternate explanation: Limit of numerator is not zero and limit of denominator is zero, so limit does not exist.	Answer 2 points

Free Response Questions: Show your work!

15. (a) State the Intermediate Value Theorem.

(b) Use the Intermediate Value Theorem to find an interval that contains a solution of the equation

$$x^3 + x + 1 = -3.$$

Essential parts of the theorem: • Let f be a continuous function on $[a, b]$ Imp. closed interval. • such that $f(a) \neq f(b)$ Imp. not presume $f(a) < f(b)$ or the other way round. • Then for any M between $f(a), f(b)$ we have a $c \in (a, b)$ such that $f(c) = M$. Imp. open interval for c.	2 points for continuous and $f(a) \neq f(b)$, 1 for M is between $f(a)$ and $f(b)$, and 2 points for the conclusion.
Define $f(x) = x^3+x+4$ and note that it is continuous everywhere, being a polynomial. Now look for a, b such that $f(a) < 0$ and $f(b) > 0$. Answer: (a, b). Alternative: $f(x) = x^3 + x + 1$. Then try for $f(a) < -3$ and $f(b) > 3$. One possible answer is $(-2, -1)$.	2 points for noting f is continuous, 2 points for the strategy of finding an interval, 1 point for making the connection with M between $f(a)$ and $f(b)$ to derive the conclusion.