MA 113 Calculus I Fall 2014
Exam 1 Tuesday, 23 September 2014

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

C, E, B, D, E, B, D, C, C, A

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the equation of the line with x-intercept 7 and parallel to the line given by the equation $4x + 3y = 9$.

   (A) $y = \frac{-3}{4}x + \frac{21}{4}$
   (B) $y = \frac{-3}{4}x + 7$
   (C) $y = \frac{-4}{3}x + \frac{28}{3}$
   (D) $y = \frac{-4}{3}x + 7$
   (E) None of the above

2. Suppose that f is a function of the form $f(x) = Ae^{kx}$, $f(1) = 5$ and $f(2) = 15$. Find A and k.

   (A) $A = 1/5$, $k = 3$
   (B) $A = 3$, $k = 3$
   (C) $A = 5$, $k = 3$
   (D) $A = 3/5$, $k = \ln(3)$
   (E) $A = 5/3$, $k = \ln(3)$

3. Suppose that $\cos(\theta) = x$ and $-\pi/2 < \theta < 0$. Find $\cos(-\theta)$ and $\sin(-\theta)$.

   (A) $\cos(-\theta) = -x$, $\sin(-\theta) = \sqrt{1-x^2}$
   (B) $\cos(-\theta) = x$, $\sin(-\theta) = \sqrt{1-x^2}$
   (C) $\cos(-\theta) = -x$, $\sin(-\theta) = -\sqrt{1-x^2}$
   (D) $\cos(-\theta) = x$, $\sin(-\theta) = -\sqrt{1-x^2}$
   (E) $\cos(-\theta) = \sqrt{1-x^2}$, $\sin(-\theta) = x$

Record the correct answer to the following problems on the front page of this exam.

4. If $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3}(f(x)+g(x)) = -2$ find

$$\lim_{x\to 3}(3f(x) - g(x)).$$

(A) 4
(B) 6
(C) 8
(D) 10
(E) 12

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} \sqrt{x}, & 0 < x < 4 \\ c, & x = 4 \\ 2\sqrt{x}+1, & x > 4 \end{cases}$$

For what choice of c is f right-continuous at $x = 4$?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Record the correct answer to the following problems on the front page of this exam.

6. The position of a particle is given by $p(t) = t^2 + t^3$. Find the average velocity of the particle on the interval $[3, 3+h]$. (Recall that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.)

   (A) $33 + 9h + h^2$

   (B) $33 + 10h + h^2$

   (C) 33

   (D) $33 - 10h + h^2$

   (E) $33 - 9h + h^2$

7. A rectangular box with no top is given. The base is a rectangle having length l meters and width w meters. The height of the box is h meters. The surface area of this box is given by

   (A) $lh + wh + lw$

   (B) lwh

   (C) $2lw + lh + wh$

   (D) $2lh + 2wh + lw$

   (E) $2lh + 2wh + 2lw$

Record the correct answer to the following problems on the front page of this exam.

8. For which of the examples below can the Intermediate Value Theorem (IVT) be used to conclude that the equation has a solution lying in the given interval?

   I. $x^3 - x = 3$, $[0, 2]$

   II. $\cos(x) - x = 0$, $[0, 1]$

   III. $2^x - 3x - 2 = 0$. $[0, 3]$

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) II and III only

9. Suppose that $f(x) = 1$, $h(x) = \sin(x)$, $g(x)$ is a function with domain $(-\infty, \infty)$, and $h(x) \leq g(x) \leq f(x)$ for all x. Then we can use the squeeze theorem to find

   (A) $\lim_{x \to 0} g(x) = 0$
   (B) $\lim_{x \to 0} g(x) = 1$
   (C) $\lim_{x \to \pi/2} g(x) = 1$
   (D) $\lim_{x \to \pi/2} g(x) = 0$
   (E) $\lim_{x \to \pi/2} g(x) = -1$

10. Find the value of the limit $\lim_{x \to 0} \dfrac{\sin(3x)\sec(x)}{2x}$.

   (A) 3/2
   (B) 2/3
   (C) 2
   (D) 3
   (E) The limit does not exist.

Free Response Questions: Show your work!

11. (a) Carefully write the definition of a function $f(x)$ that is left continuous at $x = c$.

A function $f(x)$ is left-continuous at $x = c$ if $\lim\limits_{x \to c^-} f(x) = f(c)$.

(b) Carefully draw a graph of a function $f(x)$ that satisfies the following three conditions.

i. $f(x)$ is left continuous at $x = 2$,

ii. $\lim_{x \to 2^+} f(x)$ exists,

iii. $f(x)$ is not continuous at $x = 2$.

Free Response Questions: Show your work!

12. Let $f(x) = \dfrac{2x+3}{5x-1}$.

    (a) Find the inverse function f^{-1}.

    $y = \dfrac{2x+3}{5x-1}$, $5xy - y = 2x + 3$, $x(5y-2) = y + 3$, $x = \dfrac{y+3}{5y-2}$.
    Thus $f^{-1}(x) = \dfrac{x+3}{5x-2}$.

    (b) If possible, solve $f(x) = 3$. If this is not possible, explain why.

    $f(x) = 3$, $x = f^{-1}(3) = \dfrac{3+3}{(5 \cdot 3) - 2} = \dfrac{6}{13}$.
    Thus $f(6/13) = 3$.

    (c) If possible, solve $f(x) = 2/5$. If this is not possible, explain why.

$f(x) = \dfrac{2}{5}$, $x = f^{-1}(2/5)$, but $\frac{2}{5}$ is not in the domain of $f^{-1}(x)$. Thus $f(x) = 2/5$ has no solution.
Here is another approach: $\dfrac{2x+3}{5x-1} = \dfrac{2}{5}$, $5(2x+3) = 2(5x-1)$, $15 = -2$, which is impossible.

Free Response Questions: Show your work!

13. Let $f(x) = \dfrac{1}{x}$.

(a) Find the slope of the line that passes through the points $(2, f(2))$ and $(2+h, f(2+h))$.

$$\text{Slope} = \frac{f(2+h) - f(2)}{(h+2) - 2} = \frac{\dfrac{1}{2+h} - \dfrac{1}{2}}{(h+2) - 2} = \frac{\dfrac{-h}{2(2+h)}}{h} = \frac{-1}{2(2+h)}.$$

(b) Take the limit as h tends to zero of the expression found in part a). Use the limit laws to justify your evaluation of the limit.

$$\lim_{h \to 0} \frac{-1}{2(2+h)} = \frac{-1}{2 \cdot 2} = \frac{-1}{4}.$$

(c) Give a geometric interpretation of your answer to part (b).

The slope of the tangent line to the graph of $f(x) = \dfrac{1}{x}$ at the point $x = 2$ equals $\dfrac{-1}{4}$.

Free Response Questions: Show your work!

14. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

(a) $\displaystyle\lim_{x\to -2} \frac{x^2+5x+6}{x^2-4}$.

$$\lim_{x\to -2} \frac{x^2+5x+6}{x^2-4} = \lim_{x\to -2} \frac{(x+2)(x+3)}{(x+2)(x-2)} = \lim_{x\to -2} \frac{x+3}{x-2} = \frac{-2+3}{-2-2} = \frac{1}{-4}.$$

(b) $\displaystyle\lim_{h\to 0} \frac{\sqrt{4+2h}-2}{h}$.

$$\begin{aligned}\lim_{h\to 0} \frac{\sqrt{4+2h}-2}{h} &= \lim_{h\to 0} \left(\frac{\sqrt{4+2h}-2}{h} \cdot \frac{\sqrt{4+2h}+2}{\sqrt{4+2h}+2} \right) \\ &= \lim_{h\to 0} \frac{(4+2h)-4}{h(\sqrt{4+2h}+2)} = \lim_{h\to 0} \frac{2h}{h(\sqrt{4+2h}+2)} \\ &= \lim_{h\to 0} \frac{2}{\sqrt{4+2h}+2} = \frac{2}{2+2} = \frac{1}{2}\end{aligned}$$

Free Response Questions: Show your work!

15. For each limit below, find the limit if it exists or explain why it does not exist. Carefully justify your answers.

    (a) $\lim_{h\to\frac{\pi}{2}} \frac{1-\cos(3h)}{h}$

    $$\lim_{h\to\frac{\pi}{2}} \frac{1-\cos(3h)}{h} = \frac{1-\cos\left(\frac{3\pi}{2}\right)}{\frac{\pi}{2}} = \frac{1-0}{\frac{\pi}{2}} = \frac{2}{\pi}$$

    (b) $\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\sin(\theta)}$

    $$\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\sin(\theta)} = \lim_{\theta\to 0} \frac{\sin(\theta)}{1+\cos(\theta)} = \frac{0}{1+0} = 0$$

    Here is a second approach: $\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\sin(\theta)} = \lim_{\theta\to 0} \left(\frac{1-\cos(\theta)}{\theta} \cdot \frac{\theta}{\sin(\theta)}\right) = 0 \cdot 1 = 0$

    Here is a third approach:

    $$\begin{aligned}\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\sin(\theta)} &= \lim_{\theta\to 0} \left(\frac{1-\cos(\theta)}{\sin(\theta)} \cdot \frac{1+\cos(\theta)}{1+\cos(\theta)}\right) \\ &= \lim_{\theta\to 0} \frac{\sin^2(\theta)}{\sin(\theta)(1+\cos(\theta))} = \lim_{\theta\to 0} \frac{\sin(\theta)}{1+\cos(\theta)} = \frac{0}{1+0} = 0\end{aligned}$$