Solutions

MA 113 Calculus I Fall 2016
Exam 1 Tuesday, September 20, 2016

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. (a) Find all solutions to the equation $\ln(15x) - 2\ln(1+x) = \ln(3)$. You do not need to simplify your answer.

$$\ln(15x) - \ln((1+x)^2) = \ln(3)$$

$$\Rightarrow \quad \ln\left(\frac{15}{(1+x)^2}\right) = \ln(3)$$

$$\Rightarrow \quad \frac{15}{(1+x)^2} = 3$$

$$\Rightarrow \quad 5 = (1+x)^2$$

$$\Rightarrow \quad x = \pm\sqrt{5} - 1.$$

$\boxed{\text{Since domain } \ln \text{ is } (0,\infty), \text{ only soln is } \sqrt{5}-1.}$

(b) Suppose that $f(x) = Ae^{kx}$. If $f(0) = 30$ and $f(4) = 21$, then find A and k. You do not need to simplify your answer.

$$\boxed{A = f(0) = 30.}$$

$$21 = 30 \cdot e^{k \cdot 4}$$

$$\Rightarrow \quad \frac{21}{30} = e^{k \cdot 4}$$

$$\Rightarrow \quad \ln\left(\frac{21}{30}\right) = 4k$$

$$\Rightarrow \quad \boxed{\frac{1}{4}\ln\left(\frac{21}{30}\right) = k.}$$

Free Response Questions: Show your work!

17. Evaluate the following limits, or explain why the limit does not exist. Show all your work.

(a) $\lim_{x\to\infty} \frac{7x^5 - x^4 + 2x}{\pi x^5 - 3x^3 + 1}$ $= \lim_{x\to\infty} \frac{\frac{1}{x^5}}{\frac{1}{x^5}} \cdot \frac{7x^5 - x^4 + 2x}{\pi x^5 - 3x^3 + 1}$

$$= \lim_{x\to\infty} \frac{7 - \frac{1}{x} + \frac{2}{x^4}}{\pi - \frac{3}{x^2} + \frac{1}{x^5}} = \boxed{\frac{7}{\pi}}.$$

(b) $\lim_{x\to\infty} \left[\sqrt{9x^2 + x} - 3x\right]$ $= \lim_{x\to\infty} \left[\left(\sqrt{9x^2+x} - 3x\right) \cdot \frac{\left(\sqrt{9x^2+x} + 3x\right)}{\left(\sqrt{9x^2+x} + 3x\right)}\right]$

$$= \lim_{x\to\infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2+x} + 3x} = \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3} = \boxed{\frac{1}{6}}.$$

mult. + divide by $\frac{1}{x}$

Free Response Questions: Show your work!

18. Suppose a particle has position $f(x) = x^2 - 4x$ meters at time x seconds.

(a) Find a formula for the average velocity of the particle over the time interval $[4, 4+h]$. You do not need to simplify your answer.

$$(*) = \frac{f(4+h) - f(4)}{h} = \frac{(4+h)^2 - 4(4+h) - (4^2 - 4 \cdot 4)}{h}$$

(b) Estimate the instantaneous velocity of the particle at time 4 seconds using the following three values for h: -0.1, 0.1, 0.01

Plug $-0.1, 0.1, 0.01$ into $(*)$ formula.

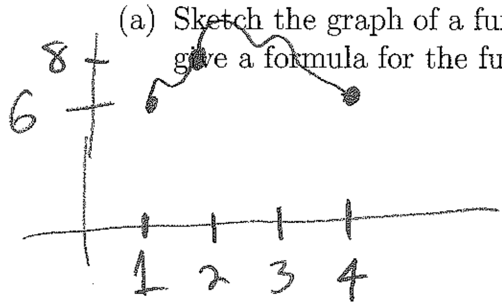
(c) Take the limit as h tends to zero of the expression found in part (a) to find the instantaneous velocity of the particle at 4 seconds. Use the limit laws to justify your evaluation of the limit.

$$\lim_{h \to 0} (*) = \lim_{h \to 0} \frac{\cancel{16} + 8h + h^2 - \cancel{16} - 4h - 0}{h} = \lim_{h \to 0} 4 + h = \boxed{4.}$$

Free Response Questions: Show your work!

19. Suppose that f is continuous on the interval $[1, 4]$ with $f(2) = 8$, and that the only solutions to $f(x) = 6$ are $x = 1$ and $x = 4$.

(a) Sketch the graph of a function that satisfies these conditions (you do not need to give a formula for the function, only sketch a graph).



(b) Use the Intermediate Value Theorem to explain why $f(3)$ must be strictly greater than 6. By assumption, $f(3) \neq 6$.

Suppose $f(3)$ was less than 6. Then f is continuous on $[2,3]$ with $f(2)=8$, $f(3)<6$. Thus, IVT implies there is some value c in $(2,3)$ with $f(c)=6$. This is not possible.