MA 113 Calculus I Fall 2016
Exam 1 Tuesday, September 20, 2016

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. True or False: For two functions f and g, the compositions $f \circ g$ and $g \circ f$ might not be equal.

2. True or False: The domain of $\arcsin(x)$ can be chosen to be $[-\pi/2, \pi/2]$.

3. True or False: If $\lim_{x\to 1} f(x) = 3$, then $f(1) \neq 3$.

4. True or False: Using only the inequalities

$$-x^2 - 1 \leq x^2 \sin(x) \leq x^2 \,,$$

the squeeze theorem can be applied to conclude that $\lim_{x\to 0} x^2 \sin(x) = 0$.

5. True or False: Every exponential function $f(x) = b^x$ with $b > 0$ has an inverse.

6. Find the equation of the line parallel to the line given by the equation $x - 7y = 15$ and passing through the point $(2, 5)$.

   (A) $y - 5 = x/7$
   (B) $y - 5 = 7(x - 2)$
   (C) $y - 5 = (x - 2)/7$
   (D) $y = 15x/2$
   (E) None of the above

7. Suppose $\lim_{x\to 0} f(x) = 4$ and $\lim_{x\to 0} g(x) = 7$. Find $\lim_{x\to 0} \dfrac{f(x)\sqrt{2 + g(x)}}{g(x)^2 - f(x)}$.

   (A) 12/9
   (B) 4
   (C) $\sqrt{2}$
   (D) 12/45
   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

8. Suppose that a function f is defined by

$$f(x) = \begin{cases} 5x - 10, & 0 < x < 3 \\ c, & x = 3 \\ x^2 - 6x + 15, & x > 3 \end{cases}$$

What value of c makes f continuous at $x = 3$?

(A) 6

(B) 5.5

(C) 3

(D) 0

(E) None of the above

9. Give the interval of continuity (including points satisfying one-sided continuity) for the function $f(x) = \dfrac{\sqrt{1-3x}}{e^x - 1}$.

(A) $(-\infty, 0) \cup (0, 1/3]$

(B) $(-\infty, 1/3]$

(C) $(1/3, \infty)$

(D) $(0, 1/3) \cup (1/3, \infty)$

(E) None of the above.

10. A bug is located at the point $(5, 0)$ at time $t = 0$ and crawls at a rate of 4 meters per minute in the counterclockwise direction along the circle centered at the origin of radius 5 meters. Find the coordinates (x, y) which give the location of the bug after t minutes.

(A) $(5\cos(4t), 5\sin(4t))$

(B) $(5\cos(4t/5), 5\sin(4t/5))$

(C) $(20\cos(4t/5), 20\sin(4t/5))$

(D) $(4\cos(t/5), 4\sin(t/5))$

(E) None of the above

Record the correct answer to the following problems on the front page of this exam.

11. Suppose that a function f is defined by

$$f(x) = \begin{cases} x+7, & 0 < x < 5 \\ 10, & x = 5 \\ x^2 - 5x + 6, & x > 5 \end{cases}$$

Let $A = \lim_{x \to 5^-} f(x)$ and $B = \lim_{x \to 5^+} f(x)$. Then $3A + 5B$ equals

(A) 66

(B) 64

(C) 80

(D) 78

(E) None of the above

12. Assume that a dish of bacteria has an initial population of 16 and doubles every hour. At what time will the population reach 1024?

(A) 10 hours

(B) 8 hours

(C) 6 hours

(D) 4 hours

(E) None of the above

13. Which of the following conditions is *not required* to apply the squeeze theorem to evaluate $\lim_{x \to a} g(x)$?

(A) $f(x) \leq g(x) \leq h(x)$ near a

(B) $\lim_{x \to a} f(x)$ exists

(C) $\lim_{x \to a} h(x)$ exists

(D) $\lim_{x \to a} g(x) = 0$

(E) All of these are required

Record the correct answer to the following problems on the front page of this exam.

14. The average velocity of a particle over the time interval $[5, 5+h]$ is given by $\dfrac{\frac{1}{5+h} - \frac{1}{5}}{h}$. The instantaneous velocity of the particle at $t = 5$ equals

    (A) 0

    (B) -1

    (C) $\dfrac{-1}{25}$

    (D) $\dfrac{-1}{5}$

    (E) None of the above

15. Evaluate $\lim\limits_{x \to 1^+} \dfrac{1}{\sin(\pi x)}$.

    (A) 1

    (B) ∞

    (C) 0

    (D) $-\infty$

    (E) The limit does not exist

16. (a) Find all solutions to the equation $\ln(15x) - 2\ln(1+x) = \ln(3)$. You do not need to simplify your answer.

(b) Suppose that $f(x) = Ae^{kx}$. If $f(0) = 30$ and $f(4) = 21$, then find A and k. You do not need to simplify your answer.

17. Evaluate the following limits, or explain why the limit does not exist. Show all your work.

(a) $\lim_{x\to\infty} \dfrac{7x^5 - x^4 + 2x}{\pi x^5 - 3x^3 + 1}$

(b) $\lim_{x\to\infty} \left[\sqrt{9x^2 + x} - 3x\right]$

18. Suppose a particle has position $f(x) = x^2 - 4x$ meters at time x seconds.

    (a) Find a formula for the average velocity of the particle over the time interval $[4, 4 + h]$. You do not need to simplify your answer.

    (b) Estimate the instantaneous velocity of the particle at time 4 seconds using the following three values for h: -0.1, 0.1, 0.01

    (c) Take the limit as h tends to zero of the expression found in part (a) to find the instantaneous velocity of the particle at 4 seconds. Use the limit laws to justify your evaluation of the limit.

19. Suppose that f is continuous on the interval $[1, 4]$ with $f(2) = 8$, and that the only solutions to $f(x) = 6$ are $x = 1$ and $x = 4$.

    (a) Sketch the graph of a function that satisfies these conditions (you do not need to give a formula for the function, only sketch a graph).

    (b) Use the Intermediate Value Theorem to explain why $f(3)$ must be strictly greater than 6.