## Name:

$\qquad$

Section: $\qquad$

Last 4 digits of student ID \#: $\qquad$
This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.
On the multiple choice problems:

- Select your answer by placing an $X$ in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.


## On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

1. Find $\sin ^{-1}\left(\cos \left(\frac{3 \pi}{4}\right)\right)$.
(A) $\frac{3 \pi}{4}$
(B) $\frac{\pi}{4}$
(C) $\frac{-3 \pi}{4}$
(D) $\frac{-\pi}{4}$
(E) None of the above
2. Suppose that $a$ and $b$ are real numbers with $a \neq 0$. If $f(x)=a x+b$, then find the inverse function $f^{-1}(x)$.
(A) $f^{-1}(x)=\frac{x+b}{a}$
(B) $f^{-1}(x)=\frac{x-b}{a}$
(C) $f^{-1}(x)=\frac{1}{a} x+b$
(D) $f^{-1}(x)=\frac{1}{a} x-b$
(E) $f^{-1}(x)=\frac{1}{a} x+\frac{1}{b}$

Record the correct answer to the following problems on the front page of this exam.
3. Find the domain of $f(x)=\sqrt{\ln (x+4)}$.
(A) $x \geq-4$
(B) $x>-4$
(C) $x \geq-3$
(D) $x \geq 0$
(E) $x>0$
4. A man of height 1.75 meters is standing 12 meters away from a 4 -meter lamppost. Find the length of the man's shadow.
(A) $8 \frac{1}{3}$ meters
(B) $8 \frac{2}{3}$ meters
(C) 9 meters
(D) $9 \frac{1}{3}$ meters
(E) $9 \frac{2}{3}$ meters

Record the correct answer to the following problems on the front page of this exam.
5. Let $c$ be a nonzero real number. Solve the equation $3 \ln (c x)-2 \ln (x)=5$.
(A) $x=e^{5}-c^{3}$
(B) $x=\frac{e^{5}}{c^{3}}$
(C) $x=5-3 \ln c$
(D) $x=e^{5}-3 \ln c$
(E) $x=\frac{5}{3 \ln c}$
6. The tangent line to the graph of $y=f(x)$ at $x=4$ is parallel to the line given by $6 x+y=5$. The tangent line intersects the $x$-axis at $x=-2$. Find the equation of the tangent line.
(A) $y=-6 x-2$
(B) $y=6 x+12$
(C) $y=6(x-4)$
(D) $y=-6 x-12$
(E) $y=-6(x-4)$

Record the correct answer to the following problems on the front page of this exam.
7. Suppose that $\lim _{x \rightarrow 3}(4 f(x)-5 g(x))=1$ and $\lim _{x \rightarrow 3} 2 g(x)=6$.

Find the value of $\lim _{x \rightarrow 3} f(x)$.
(A) 1
(B) 2
(C) 3
(D) 4
(E) None of the above
8. Suppose that

$$
f(x)= \begin{cases}\frac{(x+5)^{2}-25}{2 x}, & \text { if } x \neq 0 \\ c, & \text { if } x=0\end{cases}
$$

If $f(x)$ is continuous at all real numbers, what is the value of $c$ ?
(A) 4
(B) 4.5
(C) 5
(D) 5.5
(E) 6

Record the correct answer to the following problems on the front page of this exam.
9. Suppose that $f$ is a continuous function on the interval $[0,5]$ and we know that

$$
f(0)=1, f(1)=-1, f(2)=1, f(3)=-1, f(4)=1, \text { and } f(5)=-1
$$

Which of the following statements are true for any such $f$ ?
(A) There are at least five solutions of the equation $f(x)=0$ in the interval $[0,5]$.
(B) There are at most five solutions of the equation $f(x)=0$ in the interval $[0,5]$.
(C) There are exactly five solutions of the equation $f(x)=0$ in the interval $[0,5]$.
(D) The equation $f(x)=-1$ has at most three solutions in the interval $[0,5]$.
(E) The equation $f(x)=1$ has exactly three solutions in the interval $[0,5]$.
10. Suppose that the position of a particle at time $t$ seconds is $p(t)=t^{2}-4 t+\sqrt{t}$ meters to the right of the origin. Find the average velocity of the particle on the interval $[1,4]$.
(A) $-\frac{1}{3}$ meters/second
(B) $\frac{1}{3}$ meters/second
(C) $\frac{2}{3}$ meters/second
(D) 1 meter/second
(E) $\frac{4}{3}$ meters/second

Record the correct answer to the following problems on the front page of this exam.
11. Find the value of $c$ that makes $f(x)$ continuous at $x=4$, where

$$
f(x)= \begin{cases}5-2 x & x<4 \\ c x+7 & x \geq 4\end{cases}
$$

(A) $-\frac{1}{2}$
(B) $-\frac{3}{2}$
(C) $-\frac{5}{2}$
(D) $-\frac{7}{2}$
(E) $-\frac{9}{2}$
12. Let $f(x)= \begin{cases}5-2 x & x<4 \\ 3 x+7 & x \geq 4 .\end{cases}$

Which of the following statements below is correct?
(A) $\lim _{x \rightarrow 4^{-}} f(x)=-3$
(B) $\lim _{x \rightarrow 4^{-}} f(x)=19$
(C) $f$ is left continuous at $x=4$
(D) $f$ is not right continuous at $x=4$
(E) None of the above
13. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)
(a) $\lim _{t \rightarrow 0} \frac{2 t}{|t|}$
(b) $\lim _{x \rightarrow 0} \frac{x^{2}+3 x}{x e^{2 x}}$
14. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)
(a) $\lim _{x \rightarrow 0} \frac{\sqrt{x^{2}+64}-8}{x^{2}}$
(b) $\lim _{x \rightarrow 3} \frac{x^{2}-7 x+12}{x^{2}-x-6}$
15. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)
(a) $\lim _{x \rightarrow \infty} \frac{\sqrt{5 x^{4}-2 x^{2}+24}}{2 x^{2}-5 x+8}$
(b) $\lim _{x \rightarrow 2} \frac{x^{3}-8}{x-2}$ (Use the identity $a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$. .)
16. Consider the function $f(x)=1 /(x+2)$.
(a) Write an expression for the slope of the secant line that passes through the point $(x, f(x))$ and $(-1, f(-1))$.
(b) Take the limit as $x$ approaches -1 of the expression you found in part (a) to find the slope of the tangent line to the graph of $f$ at $x=-1$.
(c) Write the equation of the tangent line to the graph of $f$ at $x=-1$.

