MA 113 — Calculus I Spring 2008
Exam 1 February 5, 2008

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.
You are to answer *two of the last three questions*. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		6
2		10
3		8
4		9
5		9
6		8
7		9
8		10
9		14
10		14
11		14
Free	3	3
		100

(1) Consider the functions $f(x) = \dfrac{1}{x}$ and $g(x) = 4 - x^2$.

(a) Determine $f(g(1))$ and $g(f(1))$.

(b) Find all numbers a such that $(f \circ g)(a)$ is defined.

(a) $f(g(1)) =$ ____________, $g(f(1)) =$ ____________

(b) $f \circ g$ is defined on: ________________________

(2) Consider the rational function

$$f(x) = \frac{x+1}{x^2 - 2x - 3}.$$

(a) Use the limit rules to determine each of the following limits if it exists:

(i) $\lim\limits_{x \to 2^+} f(x)$

(ii) $\lim\limits_{x \to -1} f(x)$

(iii) $\lim\limits_{x \to 3^-} f(x)$

(b) Which of the lines $x = 2,\ x = -1,\ x = 3$ are vertical asymptotes of the function f?

(i) $\lim\limits_{x \to 2^+} f(x) =$ __________ (ii) $\lim\limits_{x \to -1} f(x) =$ __________ (iii) $\lim\limits_{x \to 3^-} f(x) =$ __________

(b) Vertical asymptote(s) are: ____________

(3) Let f and g be two functions such that the following limits exist:

$$\lim_{x\to 10} g(x) = 5 \quad \text{and} \quad \lim_{x\to 10}[f(x) - 2g(x)] = -7.$$

Use the limit rules to show that the following limits exist and to calculate their value:

(a) $\displaystyle\lim_{x\to 10} \frac{x}{g(x)}$.

(b) $\displaystyle\lim_{x\to 10} f(x)$.

(a) $\displaystyle\lim_{x\to 10} \frac{x}{g(x)} =$ ______________________

(b) $\displaystyle\lim_{x\to 10} f(x) =$ ______________________

(4) Consider the function $f(x) = (\frac{1}{\sqrt{x}} - 2)^{99}$.

(a) Determine the domain of f.

(b) Find all numbers a so that the function f is continuous at a.

(c) Determine $\lim_{x \to 1} f(x)$.

(a) The domain of f is ____________________

(b) f is continuous on ____________________

(c) $\lim_{x \to 1} f(x) =$ ____________________

(5) Let c be a number and consider the function

$$f(x) = \begin{cases} cx^2 - 5 & \text{if } x < 1 \\ 10 & \text{if } x = 1 \\ \dfrac{1}{x} - 2c & \text{if } x > 1 \end{cases}$$

(a) Find all numbers c such that the limit $\lim_{x \to 1} f(x)$ exists.

(b) Is there a number c such that f is continuous at 1? As always, justify your answer.

(a) $c =$ ____________ (b) yes / no (*circle the correct answer*)

(6) A particle is moving in one direction along the x-axis so that its position in meters is given by $x(t) = t^2 + 3t$ after t seconds.

(a) Find the distance the particle traveled between 1 and 2 seconds.

(b) Determine the average velocity of the particle between 2 and 4 seconds.

(a) The distance is ____________ (b) The average velocity is ____________

(7) Consider the function $f(x) = \dfrac{1}{x+2}$.

(a) Determine (as a function of a) the slope m_a of the secant line through the points $(2, f(2))$ and $(a, f(a))$ with $a \neq 2$ and $a \neq -2$. As always, simplify your answer.

(b) Find $\lim_{a \to 2} m_a$ if it exists.

(a) $m_a =$ ____________________ (b) $\lim_{a \to 2} m_a =$ ____________

(8) Using the definition, find the equation of the tangent line to the graph of the function $f(x) = \sqrt{x+3}$ at $x = 1$. Write your result in the form $y = mx + b$.

The equation of the tangent is ______________________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(9) (a) State the principle of mathematical induction. Use complete sentences.

(b) Prove by mathematical induction that, for all integers $n \geq 1$, the following equality is true:

$$\sum_{k=1}^{n}(6k+3) = 3n^2 + 6n.$$

(10) (a) State the Intermediate Value Theorem. Use complete sentences.

(b) Explain why and how you can use this theorem to show that the equation

$$x^5 - 6x - 2 = 0$$

has a root between -1 and 0.

(11) (a) State the definition of the derivative of a function f at a point a. Use complete sentences.

(b) Using the definition, determine the derivative of the function

$$f(x) = \begin{cases} 0 & \text{if } x \leq 3 \\ \sqrt{x^2 - 9} & \text{if } x > 3 \end{cases}$$

at 5 and 3 if it exists.

(b) (i) $f'(5) =$ ______________ (ii) $f'(3) =$ ______________