MA 113 — Calculus I Spring 2009
Exam 1 February 10, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number on the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		8
2		7
3		12
4		8
5		8
6		12
7		10
8		16
9		16
10		16
Free	3	3
		100

(1) Consider the functions $f(x) = 4^x$ and $g(x) = \dfrac{1}{x-2}$.

(a) Determine $f(g(1))$ and $g(f(1))$.

(b) Find all numbers a such that $(g \circ f)(a)$ is defined.

(a) $f(g(1)) =$ ________________, $g(f(1)) =$ ________________

(b) $g \circ f$ is defined on: ____

(2) Consider the function $f(x) = \dfrac{4x+1}{3x-2}$. Determine the inverse function of f.

$f^{-1}(x) =$ ________________

(3) Compute the following limits or show that they do not exist:

(a) $\displaystyle\lim_{x\to 4} \frac{2^x - 8}{\sqrt{x}+2}$, (b) $\displaystyle\lim_{h\to 0} \frac{(h+2)^2 - 4}{h}$, (c) $\displaystyle\lim_{x\to 2} \frac{x+2}{x-2}$.

(a) $\displaystyle\lim_{x\to 4} \frac{2^x - 8}{\sqrt{x}+2} =$ __________ (b) $\displaystyle\lim_{h\to 0} \frac{(h+2)^2 - 4}{h} =$ __________ (c) $\displaystyle\lim_{x\to 2} \frac{x+2}{x-2} =$ __________

(4) Let f be a function such that $\lim_{x\to 2} f(x)$ exists and $\lim_{x\to 2}\left(\frac{1}{x^2-1}f(x)+4x-1\right)=9$.
Use the limit rules to determine $\lim_{x\to 2} f(x)$. Explain your reasoning.

$\lim_{x\to 2} f(x) =$ ____________________

(5) Let f be a function such that, for all real numbers x near 1,

$$-2^{-x} + 2x + \frac{1}{2} \leq f(x) \leq x^5 + \log_2 x + 1.$$

Argue that $\lim_{x\to 1} f(x)$ exists and find its value. As usual, justify your answer.

$\lim_{x\to 1} f(x) =$ ____________________

(6) A particle is moving on a straight line so that its position after t seconds is given by $s(t) = 4t^2 - t$ meters.

(a) Find the average velocity of the particle over the time interval $[1, 2]$.

(b) Determine the average velocity of the particle over the time interval $[1, t]$, where $t > 2$. Simplify your answer.

(c) Find the instantaneous velocity of the particle at time $t = 2$.

(a) average velocity over $[1, 2]$: ____________________ m/s

(b) average velocity over $[2, t]$: ____________________ m/s

(c) instantaneous velocity at time $t = 1$: ____________________ m/s

(7) Using the definition, find the equation of the tangent line to the graph of the function $f(x) = x^2 - 4x$ at $x = 3$. Write your result in the form $y = mx + b$.

The equation of the tangent is ______________________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) Define what it means for a function f to be continuous at a. Use complete sentences.

(b) Let c be a number and consider the function

$$f(x) = \begin{cases} c2^x - 3 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ \dfrac{1}{x} - 2c & \text{if } x > 1 \end{cases}$$

Find all numbers c such that the limit $\lim_{x\to 1} f(x)$ exists.

(c) Is there a number c such that the function f in part (b) is continuous at 1? As always, justify your answer.

(b) $c =$ ______________ (c) yes / no (*circle the correct answer*)

(9) (a) State the Intermediate Value Theorem. Use complete sentences.

(b) Explain why and how you can use this theorem to show that the equation

$$2^x + x^5 + 3x + 1 = 0$$

has a solution strictly between -1 and 0.

(10) (a) State the definition of the derivative of a function f at a point a. Use complete sentences.

(b) Using the definition, determine the derivative of the function

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ \dfrac{1}{2x+1} & \text{if } x \geq 0 \end{cases}$$

at 1 and 0 if it exists.

(b) $f'(1) =$ ____________ $f'(0) =$ ____________