MA 113 Calculus I Spring 2016
Exam 1 Tuesday, February 9, 2016

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the equation of the line parallel to the line given by the equation $2x - 9y = 10$ and passing through the point $(3, -4)$.

   (A) $y = \frac{2}{9}x - \frac{10}{9}$

   (B) $y + 4 = \frac{9}{2}(x - 3)$

   (C) $2x - 9y = 42$

   (D) $y - 4 = \frac{2}{9}(x + 3)$

   (E) $y = \frac{-9}{2}x + \frac{19}{2}$

2. Consider the graph of $f(x) = 10 - x^2$. A rectangle is drawn under the graph of f in the first and second quadrants so that the sides are parallel to the x-axis and y-axis. Assume that two of the vertices of the rectangle are $(a, 0)$ and $(-a, 0)$ and that the other two vertices of the rectangle are $(a, f(a))$ and $(-a, f(-a))$, where $a > 0$ and $f(a) > 0$. The area of this rectangle equals

   (A) $20a - a^3$

   (B) $10a - 2a^3$

   (C) $10a - a^3$

   (D) $20a - 2a^3$

   (E) None of the above.

3. Find $\lim_{x \to 2} \sqrt{x + 14} \; + \lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

   (A) 6

   (B) 8

   (C) 9

   (D) 10

   (E) 12

Record the correct answer to the following problems on the front page of this exam.

4. If $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 3}(f(x)g(x)) = 40$, then find

$$\lim_{x\to 3}(3f(x) + \sqrt[3]{g(x)} + g(x)^2).$$

(A) 84

(B) 83

(C) 82

(D) 81

(E) 80

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} 7x - 12, & 0 < x < 4 \\ c, & x = 4 \\ 2x^2 - 5x + 2, & x > 4 \end{cases}$$

For what choice of c is f right-continuous at $x = 4$?

(A) 18

(B) 17

(C) 16

(D) 15

(E) 14

Record the correct answer to the following problems on the front page of this exam.

6. Suppose that a function f is defined by

$$f(x) = \begin{cases} 2x + 5, & 0 < x < 3 \\ 14, & x = 3 \\ x^2 - 5x + 4, & x > 3 \end{cases}$$

Let $A = \lim_{x \to 3^-} f(x)$ and $B = \lim_{x \to 3^+} f(x)$. Then $4A + 7B$ equals

(A) 30

(B) 31

(C) 32

(D) 33

(E) None of the above

7. Assume that f is a continuous function with domain the set of all real numbers. Suppose that $f(-2) = -7$, $f(0) = 2$, $f(2) = -3$, $f(4) = 5$, $f(6) = -1$, $f(x) < 0$ for $x \leq -2$, and $f(x) < 0$ for $x \geq 6$. The Intermediate Value Theorem lets us conclude which of the following statements.

(A) The equation $f(x) = 0$ has exactly 4 solutions.

(B) The equation $f(x) = 0$ has at least 4 solutions.

(C) The equation $f(x) = 0$ has at most 4 solutions.

(D) The equation $f(x) = 0$ might have no solutions.

(E) None of the above statements can be justified with the given information.

Record the correct answer to the following problems on the front page of this exam.

8. Given that $(x - a^2) = (\sqrt{x} + a)(\sqrt{x} - a)$, find $\lim_{x \to a^2} \dfrac{\sqrt{x} - a}{x - a^2}$ for an arbitrary positive real number a.

   (A) $2a$
   (B) a
   (C) $\frac{1}{a}$
   (D) $\frac{1}{2a}$
   (E) The limit does not exist.

9. The average velocity of a particle over the time interval $[5, 5+h]$ is given by $\dfrac{7h^2 + 16h}{h(8-h)^2}$. The instantaneous velocity of the particle at $t = 5$ equals

   (A) $\dfrac{1}{2}$
   (B) $\dfrac{1}{4}$
   (C) $\dfrac{1}{8}$
   (D) $\dfrac{1}{16}$
   (E) $\dfrac{1}{64}$

10. Find the average rate of change of the function $f(x) = x^3 + 1$ between the points $(2, f(2))$ and $(5, f(5))$.

    (A) 36
    (B) 37
    (C) 38
    (D) 39
    (E) 40

11. (a) Find all solutions to the equation $2\ln(x) - \ln(6 - x) = \ln(3)$.

(b) Suppose that $f(x) = Qe^{kx}$. If $f(0) = 20$ and $f(5) = 40$, then find Q and k.

12. Let $f(x) = \dfrac{3x-4}{4x-5}$.

(a) Find the domain of f.

(b) Find the inverse function f^{-1}.

(c) Find the domain of f^{-1}.

(d) If possible, solve $f(x) = 3$. If this is not possible, explain why.

(e) If possible, solve $f(x) = \frac{3}{4}$. If this is not possible, explain why.

13. Let $f(x) = x^2 - 2x$.

(a) Find the slope of the line that passes through the points $(4, f(4))$ and $(4 + h, f(4 + h))$.

(b) Take the limit as h tends to zero of the expression found in part (a) to find the slope of the tangent line to the graph of $f(x)$ at $x = 4$. Use the limit laws to justify your evaluation of the limit.

(c) Write the equation of the tangent line to the graph of $f(x)$ at $x = 4$ in point-slope form.

14. For each problem below, find the limit or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x \to 3} \dfrac{x^3 - 5x^2 + 6x}{x - 3}$

(b) $\lim_{x \to \frac{\pi}{2}} \dfrac{1 - \sin(x)}{1 - \sin^2(x)}$

15. For each problem below, find the limit or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{h \to 0} \dfrac{\tan(3h)}{2h}$

(b) $\lim_{x \to 1} \left(\dfrac{1}{x-1} + 2 \right)$