MA 113 Calculus I Spring 2017
Exam 1 Tuesday, February 7, 2017

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. True or False: There are three functions f, g, and h that allow us to write $3x^2 + 7$ as $f \circ g \circ h$.

2. True or False: The domain of $\arccos(x)$ can be chosen to be $[-\pi/2, \pi/2]$.

3. True or False: Using only the inequalities

$$-x^4 \leq x^4 \sin(1/x) \leq x^4 \,,$$

the squeeze theorem can be applied to conclude that $\lim_{x\to 0} x^4 \sin(1/x) = 0$.

4. True or False: If $\lim_{x\to 5} f(x) = 2$, then $f(5) \neq 2$.

5. True or False: If the average velocity of a particle between 2 seconds and 2.05 seconds is 4.3 meters per second, then the instantaneous velocity of the particle at 2 seconds is 4.25 meters per second.

6. Find the equation of the line passing through the point $(1, 3)$ that is perpendicular to the line given by the equation $-2x + y = 6$.

   (A) $y - 3 = \frac{1}{2}(x - 1)$
   (B) $y - 1 = \frac{-1}{2}(x - 3)$
   (C) $y = 6x$
   (D) $y - 3 = \frac{-1}{2}(x - 1)$
   (E) None of the above

7. Suppose $\lim_{x\to 1} f(x) = 3$ and $\lim_{x\to 1} g(x) = 5$. Find $\lim\limits_{x\to 1} \dfrac{f(x)g(x) - 2}{g(x)\sqrt{f(x) - 2}}$.

   (A) 13/5
   (B) 3
   (C) 13/10
   (D) 5
   (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

8. Give the intervals of continuity for $\dfrac{x+1}{x^2+4x+3}$.

   (A) $(-\infty, 1) \cup (1, 3) \cup (3\infty)$

   (B) $(-\infty, -3) \cup (-3, 1) \cup (1\infty)$

   (C) (∞, ∞)

   (D) $(-\infty, -3) \cup (-3, -1) \cup (-1\infty)$

   (E) None of the above

9. Suppose that

$$f(x) = \begin{cases} (x-5)/|x-5| & \text{if } x \neq 5 \\ 1 & \text{if } x = 5 \end{cases}.$$

   Determine the values of x for which $f(x)$ is not continuous and state the type of discontinuity.

   (A) $x = 5$, removable discontinuity

   (B) $x = 5$, jump discontinuity

   (C) $x = 1$, jump discontinuity

   (D) $x = 1$ and $x = 5$, both jump discontinuities

   (E) None of the above.

10. A bug is located at the point $(8, 0)$ at time $t = 0$ and crawls at a rate of 3 meters per minute in the counterclockwise direction along the circle centered at the origin of radius 8 meters. Find the coordinates (x, y) which give the location of the bug after t minutes.

    (A) $(8\cos(3t), 8\sin(3t))$

    (B) $(8\cos(3t/8), 8\sin(3t/8))$

    (C) $(24\cos(3t/8), 24\sin(3t/8))$

    (D) $(3\cos(t/8), 3\sin(t/8))$

    (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

11. Suppose that a function f is defined by

$$f(x) = \begin{cases} x+7, & 0 < x < 5 \\ 7, & x = 5 \\ x^2 - 5x + 4, & x > 5 \end{cases}$$

Let $A = \lim_{x \to 5^-} f(x)$ and $B = \lim_{x \to 5^+} f(x)$. Then $5A + 3B$ equals

(A) 12

(B) 47

(C) 56

(D) 72

(E) None of the above

12. Assume that a dish of bacteria has an initial population of 12 and doubles every hour. At what time will the population reach 192?

(A) 6 hours

(B) 5 hours

(C) 4 hours

(D) 3 hours

(E) None of the above

13. Which of these functions, if any, is not continuous at 3?

(A) $(x-3)^2$

(B) $\cos(\sin(x/3))$

(C) $e^{\frac{1}{3x}}$

(D) $\dfrac{x-3}{x+3}$

(E) All of these functions are continuous at 3

Record the correct answer to the following problems on the front page of this exam.

14. The average velocity of a particle over the time interval $[7, 7+h]$ is given by $\frac{\frac{1}{7+h} - \frac{1}{7}}{h}$. The instantaneous velocity of the particle at $t = 7$ equals

    (A) $\frac{1}{7}$

    (B) $\frac{-1}{49}$

    (C) $\frac{-1}{7}$

    (D) $\frac{1}{49}$

    (E) None of the above

15. Suppose that the instantaneous velocity of a particle at x seconds is given by

$$\lim_{h \to 0} \frac{\cos(x^2 + 2xh + h^2) - \cos(x^2)}{h}$$

    for all real numbers x. What is the function giving the position of the particle at x seconds?

    (A) $\cos(x^2)$

    (B) x^2

    (C) $\cos(2x)$

    (D) $\cos(x^2) - \cos(x)$

    (E) None of the above

16. (a) Find all solutions to the equation $2\ln(x) - \ln(1+x) = \ln(1)$.

(b) Suppose that $f(x) = Ae^{kx}$. If $f(0) = 20$ and $f(3) = 17$, then find A and k. You do not need to simplify your answer.

17. Evaluate the following limits, or explain why the limit does not exist. Show all your work.

(a) $\lim_{x\to\infty} \dfrac{3x^4 - x^2 + x - \pi}{7x^4 - 3x^3 + 1}$

(b) $\lim_{x\to\infty} \left[\sqrt{4x^2 + 1} - 2x\right]$

18. Suppose a particle has position $f(x) = 3x^2 - x$ meters at time x seconds.

    (a) Find a formula for the average velocity of the particle over the time interval $[5, 5 + h]$. You do not need to simplify your answer.

    (b) Estimate the instantaneous velocity of the particle at time 5 seconds using the following three values for h: -0.1, 0.1, 0.01

    (c) Take the limit as h tends to zero of the expression found in part (a) to find the instantaneous velocity of the particle at 5 seconds. Use the limit laws to justify your evaluation of the limit.

19. (a) State the intermediate value theorem.

(b) If $g(x) = x^2 + 5^x - 3$, use the intermediate value theorem to show that there is a number a such that $g(a) = 10$.