MA 113 Calculus I

Spring 2019

Exam ~~[illegible]~~ 1

Tuesday, 5 ~~March~~ Feb 2019

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

#14 KEY * #14 KEY * #14 KEY * #14 KEY

Free Response Questions: Show your work!

(5 pts) 14. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x\to 2} \frac{x^3-8}{x-2}$

$$\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$$ (+1) factoring

$$= \lim_{x\to 2} (x^2+2x+4)$$ (+2) writing equivalent limit

$$= (2)^2 + 2(2) + 4 = \boxed{12}$$

(+1) plug in 2 (+1) answer

(5pts) (b) $\lim_{x\to\pi/2} \cos(x)\cos(\tan(x))$

(+1) Since $-1 \le \cos(x) \le 1$, we can say $|\cos(x)| \le 1$
also $|\cos(\tan(x))| \le 1$

(+1) then $|\cos(x)\cos(\tan(x))| \le |\cos(x)|$
OR $-|\cos(x)| \le \cos(x)\cos(\tan(x)) \le |\cos(x)|$

(+1) So $\lim_{x\to\pi/2} -|\cos(x)| \le \lim_{x\to\pi/2} \cos(x)\cos(\tan(x)) \le \lim_{x\to\pi/2} |\cos(x)|$

(+1) yields $0 \le \lim_{x\to\pi/2} \cos(x)\cos(\tan(x)) \le 0$

(+1) Thus, by the Squeeze Theorem,
$$\lim_{x\to\pi/2} \cos(x)\cos(\tan(x)) = \boxed{0}$$

4/5 pts if
$-\cos(x) \le \cos(x)\cos(\tan(x)) \le \cos(x)$
then followed through with Squeeze Thm to get
$\lim_{x\to\pi/2} \cos(x)\cos(\tan(x)) = 0$

Potential Special Cases:

- Correct use of "highest power rule": 2/5
- Table of values: 2/5?
- Mult by $\frac{1/x^3}{1/x^3}$ in (a) w/ correct expl. 5/5

Free Response Questions: Show your work!

15. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x\to\infty} \frac{3x^3-7x+2}{\pi x^2+14}$ $= \lim_{x\to\infty} \frac{1/x^2\,(3x^3-7x+2)}{1/x^2\,(\pi x^2+14)}$ ① mult $\frac{1/x^2}{1/x^2}$

$= \lim_{x\to\infty} \frac{3x - 7/x + 2/x^2}{\pi + 14/x^2}$ ① simp.

$\lim_{x\to\infty} 3x - 7/x + 2/x^2 = \infty$, and

$\lim_{x\to\infty} \pi + 14/x^2 = \pi$,

Since the denominator is approaching a constant while the numerator increases without bound,

② expl.

$\lim_{x\to\infty} \frac{3x - 7/x + 2/x^2}{\pi + 14/x^2} = \boxed{\infty}$ ① ans.

/5

(b) $\lim_{x\to-\infty} \frac{\sqrt{2x^{10}+3x}}{6x^5}$ $= \lim_{x\to-\infty} \frac{1/x^5\,\sqrt{2x^{10}+3x}}{1/x^5 \cdot 6x^5}$ ① mult $\frac{1/x^5}{1/x^5}$

$= \lim_{x\to-\infty} \frac{-\sqrt{1/x^{10}}\cdot\sqrt{2x^{10}+3x}}{1/x^5\cdot 6x^5}$ because $x<0$ ① $1/x^5 = -\sqrt{1/x^{10}}$

$= \lim_{x\to-\infty} \frac{-\sqrt{2+3/x^9}}{6}$ ① simp.

$= \frac{\lim_{x\to-\infty} -\sqrt{2+3/x^9}}{\lim_{x\to-\infty} 6} = \frac{-\sqrt{\lim_{x\to-\infty} 2 + \lim_{x\to-\infty} 3/x^9}}{\lim_{x\to-\infty} 6}$ ① limit laws

$= \frac{-\sqrt{2+0}}{6} = \frac{-\sqrt{2}}{6}$ ① ans.

/5

16. (a) State the Intermediate Value Theorem. ① ①.

Assume $f(x)$ is continuous on $[a,b]$.

① If N is a number between $f(a)$ and $f(b)$ then

there is a c with $a < c < b$ ①

s.t. $f(c) = N$. ①

(b) Let $f(x) = x^5 - x^4 + 2x^2 - \frac{1}{4}$. Use the Intermediate Value Theorem to show that there must exist a solution to $f(x) = \frac{1}{2}$ in the interval $[0, 1]$.

① $f(0) = -1/4$

① $f(1) = 1 - 1 + 2 - 1/4$

$= 1\,3/4 \; (= 7/4)$.

① f is continuous on $[0,1]$ since f is a polynomial.

① So Thus there is a ~~z~~ with $0<z<1$ s.t. $f(z) = 1/2$

① $f(0) = -1/4 < 1/2 < 7/4 = f(1)$

Free Response Questions: Show your work!

13. Assume that the position of an object after t seconds is given by $f(t) = 10t^3 + 1$ meters.

(a) Write an expression for the average velocity of the object on the interval $[1, 1+h]$. Include units!

$$\frac{\Delta f}{\Delta t} = \frac{f(1+h)-f(1)}{\underbrace{1+h-1}} = \frac{10(1+h)^3+1-(10+1)}{\underbrace{h}}$$

either is (+2)

If this algebra does not appear until (c), no problem.

$$= \frac{10(1+h)^3 - 10}{h}$$

$$= \boxed{30 + 30h + 10h^2} \; \boxed{m/s}$$

(+1) (+1)

(b) Compute the average velocity over the time intervals $[0.999, 1]$ and $[1, 1.001]$ to estimate the instantaneous velocity. Include units!

$$30 + 30(-.001) + 10(-.001)^2 = 29.98 \; m/s$$

(+1) (+1)

$$30 + 30(0.001) + 10(0.001)^2 = 30.04 \; m/s$$

(+1)

If one of these units is missing, the point is lost.

anything equivalent to this formula is fine.

(c) Take the limit as h approaches 0 of the expression you found in part (a) to find the instantaneous velocity of the object at time $t = 1$ seconds. Include units!

$$\underbrace{\lim_{h \to 0} 30 + 30h + 10h^2}_{(+1)} = 30 + 30 \cdot 0 + 10 \cdot 0^2$$

$$= 30 \; m/s$$

(+1) (+1)