

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. This is a two-hour exam.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

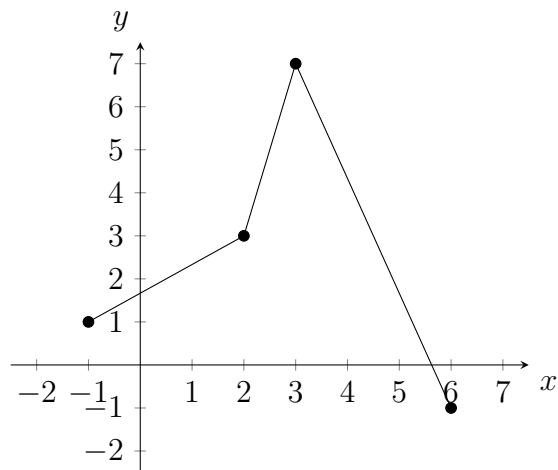
| Question | | | | | |
|----------|---|---|---|---|---|
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |

Exam Scores

| Question | Score | Total |
|----------|-------|-------|
| MC | | 60 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| 16 | | 10 |
| Total | | 100 |

1. What are the domain and range of $f(x)$ found in the graph below?

- (A) Domain: $[1, 6]$ Range: $[1, 7]$
- (B) Domain: $[-1, 6]$ Range: $[1, 7]$
- (C) Domain: $[-1, 6]$ Range: $[-1, 7]$
- (D) Domain: $-1, 2, 3, 6$ Range: $1, 3, 7, -1$
- (E) None of the above



2. The population of a city at time t is $P(t) = 520e^{2t}$. When will the population be three times larger than $P(0)$?

- (A) $\frac{\ln(3)}{2}$
- (B) $\frac{\ln(2)}{3}$
- (C) $2\ln(3)$
- (D) $520\ln(3)$
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

3. Find $\sin(2\theta)$ if $\tan(\theta) = \frac{2}{3}$.

(A) $\frac{5}{13}$

(B) $\frac{12}{13}$

(C) $\frac{6}{13}$

(D) $\frac{4}{\sqrt{13}}$

(E) None of the above

4. Suppose the position of an object is given by $f(x) = 2x^3$ meters at time x seconds. Find the instantaneous velocity of the object at $x = 3$ seconds.

(A) 54

(B) 27

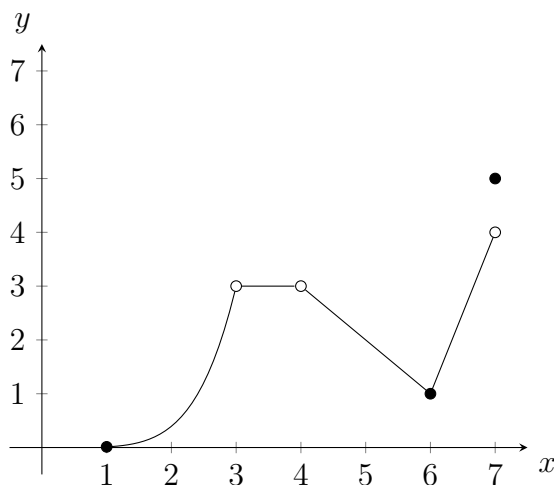
(C) 9

(D) 18

(E) None of the above

Record the correct answer to the following problems on the front page of this exam.

5. Let $f(x)$ be the function whose graph is shown below. Which of the following limits are all correct?



- (A) $\lim_{x \rightarrow 3^-} f(x) = 0$ and $\lim_{x \rightarrow 4} f(x) = 0$ and $\lim_{x \rightarrow 6^+} f(x) = 1$
 (B) $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 7^-} f(x) = 7$
 (C) $\lim_{x \rightarrow 3} f(x)$ does not exist and $\lim_{x \rightarrow 4} f(x)$ does not exist and $\lim_{x \rightarrow 7^-} f(x) = 4$.
 (D) $\lim_{x \rightarrow 3} f(x) = 3$ and $\lim_{x \rightarrow 4} f(x) = 3$ and $\lim_{x \rightarrow 7^-} f(x) = 4$.
 (E) None of the above.

6. Suppose that $f(x) = \begin{cases} x + 1, & \text{if } x \leq 2 \\ Ax + B & \text{if } 2 \leq x \leq 3.5 \\ -6x + 25 & \text{if } x \geq 3.5. \end{cases}$

Find the values of A and B which make $f(x)$ continuous everywhere.

- (A) $A = 5/3, B = 2/3$
 (B) $A = 1/3, B = 5/3$
 (C) $A = 1, B = 1$
 (D) $A = 1, B = 5/3$
 (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

7. If $\lim_{x \rightarrow 5} f(x) = 8$ and $\lim_{x \rightarrow 5} g(x) = 4$, find $\lim_{x \rightarrow 5} \frac{1 + g(x)^2}{3f(x) - g(x)}$.

- (A) $\frac{26}{10}$
- (B) $\frac{17}{19}$
- (C) $\frac{26}{11}$
- (D) $\frac{17}{20}$
- (E) Limit does not exist

8. Find the horizontal asymptotes of $f(x) = \frac{1 + e^{-x}}{5e^x}$.

- (A) $y = 0$ and $y = 1$
- (B) $y = 0$
- (C) $y = 1/5$
- (D) $y = 1$
- (E) None of the above

9. Find the positive solution to the equation $\ln(x^{10}) - \ln(x^8) = \ln 2$.

- (A) $\sqrt{e^2}$
- (B) $\sqrt{\ln(2)}$
- (C) $\sqrt{2}$
- (D) 2
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

10. Let O be the center of a circle whose circumference is 52 centimeters. Let P and Q be two points on the circle that are endpoints of an arc that is 13 centimeters long. Find the angle between the segments OP and OQ .

- (A) $\pi/2$ radians
- (B) $\pi/3$ radians
- (C) $\pi/4$ radians
- (D) $\pi/6$ radians
- (E) None of the above

11. Which of the following statements is true if $f(x)$ is defined to be

$$f(x) = \begin{cases} \sin x & \text{if } x \leq -\pi/4 \\ \cos x & \text{if } x > -\pi/4 \end{cases}$$

- (A) $f(x)$ is continuous at $x = -\pi/4$
 - (B) $f(x)$ has a jump discontinuity at $x = -\pi/4$
 - (C) $f(x)$ has an infinite discontinuity at $x = -\pi/4$
 - (D) $f(x)$ is not defined at $x = -\pi/4$
 - (E) None of the above.
12. A bug is located at the point $(12, 0)$ at time $t = 0$ and crawls at the rate of 4 units per minute in the counterclockwise direction along the circle centered at the origin of radius 12. Find the coordinates (x, y) which give the location of the bug after 5 minutes.
- (A) $(12 \cos(5/4), 12 \sin(5/4))$
 - (B) $(12 \cos(5/3), 12 \sin(5/3))$
 - (C) $(4 \cos(5/3), 4 \sin(5/3))$
 - (D) $(12 \cos(3/5), 12 \sin(3/5))$
 - (E) None of the above

Free Response Questions: Show your work!

13. Assume that the position of an object after t seconds is given by $f(t) = 10t^3 + 1$ meters.

(a) Write an expression for the average velocity of the object on the interval $[1, 1 + h]$. Include units!

(b) Compute the average velocity over the time intervals $[0.999, 1]$ and $[1, 1.001]$ to estimate the instantaneous velocity. Include units!

(c) Take the limit as h approaches 0 of the expression you found in part (a) to find the instantaneous velocity of the object at time $t = 1$ seconds. Include units!

Free Response Questions: Show your work!

14. Find the limits or state that the limit does not exist. In each case, justify your answer.

(Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

(b) $\lim_{x \rightarrow \pi/2} \cos(x) \cos(\tan(x))$

Free Response Questions: Show your work!

15. Find the limits or state that the limit does not exist. In each case, justify your answer.
(Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x \rightarrow \infty} \frac{3x^3 - 7x + 2}{\pi x^2 + 14}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^{10} + 3x}}{6x^5}$

Free Response Questions: Show your work!

16. (a) State the Intermediate Value Theorem.

(b) Let $f(x) = x^5 - x^4 + 2x^2 - \frac{1}{4}$. Use the Intermediate Value Theorem to show that there must exist a solution to $f(x) = \frac{1}{2}$ in the interval $[0, 1]$.