MA 113 — Calculus I Fall 2010
Exam 1 September 21, 2010

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		8
2		8
3		9
4		12
5		8
6		10
7		10
8		16
9		16
10		16
Free	3	3
		100

(1) Consider the functions $f(x) = \frac{1}{x-6}$ and $g(x) = x^2 + 2$.

(a) Compute $f(g(5))$ and $g(f(5))$. **Give exact answers.**

(b) Let h be the composite function $h(x) = (f \circ g)(x)$. Find the domain of h. **As usual, justify your answer by showing your work.**

(a) $f(g(5)) =$ __________, $g(f(5)) =$ __________

(b) Domain of h is ____________________

(2) (a) Solve the equation $5^{x+2} = 7$. **Show all steps of the computation and give the exact answer.**

(b) Express the quantity

$$\log_4(x^6 + 1) - \log_4(9x) + \frac{1}{5}\log_4(x)$$

as a single logarithm.

(a) Solution is ______________________________

(b) ______________________________

(3) Consider the function

$$f(x) = \frac{2x-4}{5x+1}.$$

(a) Find the domain of f.

(b) Find the inverse function f^{-1} of f.

(a) Domain of f is ______________________

(b) $f^{-1}(x) =$ ______________________

(4) Use the limit rules and continuity to determine each of the following limits if it exists. If a limit does not exist, but is ∞ or $-\infty$, then clearly indicate that.

(a) $\lim_{x\to 16} 2^{\sqrt{x}-5}$

(b) $\lim_{h\to 0} \dfrac{(h+4)^2-16}{h}$.

(c) $\lim_{x\to 4} \dfrac{x^2+1}{x-4}$.

(a) ____________________________________

(b) ____________________________________

(c) ____________________________________

(5) Let f and g be two functions such that the following limits exist

$$\lim_{x\to 2} g(x) = 7, \quad \lim_{x\to 2} \left[3^x f(x) - x g(x)\right] = 13.$$

Use the limit laws to compute the following limits.

(a) $\lim_{x\to 2} \frac{g(x)}{x^3 - 1}$.

(b) $\lim_{x\to 2} f(x)$.

(a) ________________________________

(b) ________________________________

(6) A particle is moving on a straight line so that after t seconds it is $s(t) = 5t + 1$ meters to the right of a reference point.

(a) Find the average velocity of the particle over the time interval $1 \leq t \leq 2$.

(b) Find the average velocity over the time interval $[1, t]$, where $t > 1$. **Simplify your answer.**

(c) Use your answer in (b) to find the instantaneous velocity of the particle after 1 second.

(a) Average velocity over $[1, 2]$ is ________________ $\frac{m}{s}$

(b) Average velocity over $[1, t]$ is ________________ $\frac{m}{s}$

(c) Instantaneous velocity at time $t = 1$ is ____________ $\frac{m}{s}$

(7) Using the definition, find the equation of the tangent line to the graph of the function $f(x) = x^2 - 5x$ at $x = 2$. Write your answer in the form $y = mx + b$.

Equation of the tangent line: ____________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) Define what it means for a function f to be continuous at a. Use complete sentences.

Let

$$f(x) = \begin{cases} c3^x - 16, & \text{if } x < 2, \\ 5, & \text{if } x = 2, \\ x^2 - c, & \text{if } x > 2. \end{cases}$$

As always, justify your answer to the following problems!

(b) Find all values for c such that $\lim_{x \to 2} f(x)$ exists.

(c) For which of the values for c found in (b) is the function f continuous at 2?

(d) Find all values for c such that the function f is continuous at 1.

(b) ______________ (c) ______________ (d) ______________

(9) (a) State the Intermediate Value Theorem. Use complete sentences.

(b) **Explain in detail why and how** you can use this theorem to show that the equation

$$2^x - 3\sqrt{x} = 1$$

has a solution in the interval $(1, 4)$.

(10) (a) State the definition of the derivative of a function at a point a. Use complete sentences.

(b) Using the definition, determine the derivative of the function

$$f(x) = \begin{cases} \dfrac{1}{3x+2} & \text{if } x \leq 0 \\ 7 & \text{if } x > 0. \end{cases}$$

at $x = -1$ and $x = 0$ if it exists.

$f'(-1) =$ ____________________ $f'(0) =$ ____________________