

Calculus I
Exam 2

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Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*). Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question. The total on this test is 103 points, however no student will be given a grade of higher than 100.

Name _____

Section _____

Question	Score	Total
1		10
2		24
3		5
4		5
5		10
6		5
7		10
8		10
9		8
10		8
11		8
Total		100

1. (a) State the product rule. Be sure to give the hypotheses and the conclusion.
(b) Prove the product rule.

2. For each of the following functions, find $f'(x)$. Simplify, if possible.

(a) $f(x) = x^2 + \frac{1}{x^3}$

(b) $f(x) = \frac{1 - x^3}{1 + x^3}$.

(c) $f(x) = \cos(\sin(x^2))$

(d) $f(x) = x \sin(x)$

(e) $f(x) = \sqrt{1 + \sin^2 x}$

(f) $f(x) = \frac{\cos^2 x}{\cos x \sin x}$

3. (a) Find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}.$$

(b) Find the limit

$$\lim_{x \rightarrow \pi} \frac{1 - \cos x}{x^2}.$$

4. Find a quadratic function $p(x)$ so that

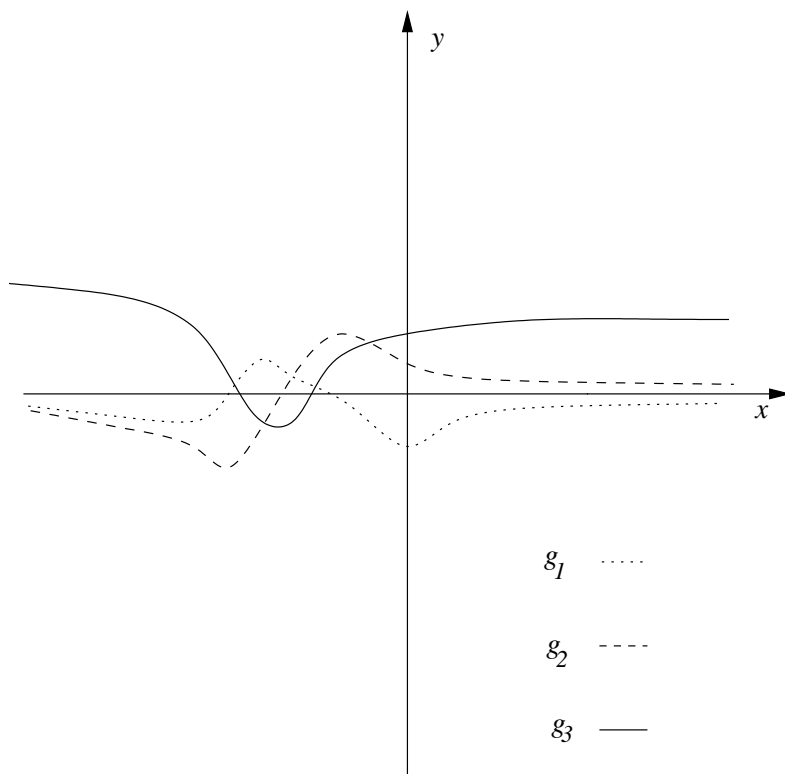
$$p(0) = 1, \quad p'(1) = 1, \quad \text{and} \quad p''(2) = 2.$$

5. Consider the ellipse which is defined by the equation

$$3x^2 + 2y^2 = 120.$$

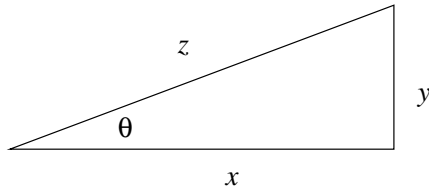
- (a) Find $\frac{dy}{dx}$.
- (b) Find all points on the ellipse with $x = 4$.
- (c) Find the equations of the tangent lines to the curve $3x^2 + 4y^2 = 120$ at the points you found in part b).

6. The graph below gives the graph of three functions, $g_1(x)$, $g_2(x)$ and $g_3(x)$ which are f , f' and f'' for some function f . Determine which of g_1 , g_2 and g_3 are f , f' and f'' . Explain your reasoning.



7. In this problem, you are to find the equations of all tangent lines to $y = x^2$ which pass through $(0, -4)$.
- (a) Find the values of a so that the tangent line to $y = x^2$ at $x = a$ passes through the point $(0, -4)$.
 - (b) Write the equations of the tangent lines to $y = x^2$ at the values of a you found in the part a).

8. (a) Suppose x , y and z are the lengths of the sides of a right triangle (see sketch). If the length z is fixed, $0 < \theta < \pi/2$ and θ is increasing, will x increase or decrease? (Here, you are only asked to guess. You should compare your guess with the answer you obtain in part b) and attempt to make a correction if your answers contradict each other.)
- (b) Suppose that in addition to the information in part a), we know $z = 20$ meters and that θ is increasing at a rate of 0.5 radians/second. Find the rate of change of x with respect to time when $\theta = \pi/6$.



9. (a) Find the linear and quadratic approximations to $f(x) = \sqrt[3]{x} = x^{1/3}$ at $x = 27$.
- (b) Use the linear and quadratic approximations in part a) to find two approximate values for $\sqrt[3]{26}$.

10. Use Newton's method to find a solution of $x^3 = 26$. Begin with an initial guess of $x_0 = 3$. Give the iterates x_1 , x_2 and x_3 rounded correctly to four decimal places. Show your work.

11. Consider the function f whose graph appears below.

- (a) If we apply Newton's method with initial guess $x_0 = A$, which of the roots r_1 , r_2 or r_3 will we find? Sketch one step of Newton's method to justify your answer.
- (b) If we apply Newton's method with initial guess $x_0 = B$, which of the roots r_1 , r_2 or r_3 will we find? Sketch one step of Newton's method to justify your answer.

