

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Last four digits of student identification number: \_\_\_\_\_

Question	Score	Total
1		10
2		8
3		13
4		10
5		10
6		8
7		8
8		15
9		15
10		15
Free	3	3
		100

(1) Find the derivative of the following functions.

(a)  $g(x) = \frac{x^3}{x^2 + 1}$ .

(b)  $h(t) = t^2 \cdot e^{t^2+1}$ .

(c)  $f(x) = \tan(3x)$

(a)  $g'(x) =$  \_\_\_\_\_

(b)  $h'(t) =$  \_\_\_\_\_

(c)  $f'(x) =$  \_\_\_\_\_

(2) Given that

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$$

use the limit laws to find the limits.

(a)  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x}$ .

(b)  $\lim_{x \rightarrow 0} \left( e^{x^3+1} \frac{\sin(4x)}{x} \right)$

(a)  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x} = \underline{\hspace{2cm}}$

(b)  $\lim_{x \rightarrow 0} \left( e^{x^3+1} \frac{\sin(4x)}{x} \right) = \underline{\hspace{2cm}}$

- (3) A particle is traveling along a straight line. Its position after  $t \geq 0$  seconds is given by

$$s(t) = \frac{1}{3}t^3 - t^2 - 3t + 1.$$

meters. **As usual, justify your answer.**

- (a) Find the time interval(s) where the particle is traveling in the positive direction?

- (b) What is the total distance traveled by the particle during the first 6 seconds?

- (c) Find the time interval(s) where the particle is speeding up?

(a) Time interval(s) \_\_\_\_\_

(b) Total distance \_\_\_\_\_

(c) Time interval(s) \_\_\_\_\_

(4) Use the differentiation rules to determine the following higher order derivatives. **As always, show your work.**

(a) Find  $f''(x)$  if  $f(x) = \ln(3x^2 + 5x - 4)$ .

(b) Find  $g''(x)$  if  $g(x) = (2x + 1) \cdot \sin(3x)$

(a)  $f''(x) =$  \_\_\_\_\_

(b)  $g''(x) =$  \_\_\_\_\_

- (5) Consider the curve described by the equation  $xy^2 + x^2y + y^3 = 7$ . Find the equation of the line tangent to this curve at the point  $(2, 1)$ . Write your answer in the form  $y = mx + b$ . **As always, show your work.**

Equation of the tangent line is \_\_\_\_\_

(6) As usual show your work in answering the following questions. **Give the exact answer!**

(a) Find  $F'(\pi/4)$  where  $F(x) = \ln |\sin(x)|$ .

(b) Find  $G'(2)$  where  $G(x) = \arctan(e^x)$

(a)  $F'(\pi/4) =$  \_\_\_\_\_

(b)  $G'(2) =$  \_\_\_\_\_

- (7) A certain bacteria culture is known to grow exponentially, that is, its population at time  $t$  is given by a function of the form

$$p(t) = p_0 e^{kt},$$

where  $k$  is a constant and  $p_0$  is the initial population. It has been found that the culture tripled in size in 10 hours. When did it double in size? **Give the exact answer.**

It doubled in \_\_\_\_\_ hours.



Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) Consider the function  $f(x) = 3x^2 + 6$ .

(a) Find the equation of the tangent line to the graph of  $f(x)$  at the point  $(a, f(a))$ .

(b) Find all points  $(a, f(a))$  on the graph of  $f(x)$  such that the tangent line to the graph at  $x = a$  passes through the point  $(2, -9)$ . **As usual, show your work to support your answer.**

(a) Equation of the tangent line: \_\_\_\_\_

(b) The points are: \_\_\_\_\_

- (9) (a) State the Chain Rule. Use complete sentences and include all assumptions necessary to make the rule valid.

(b) Suppose  $f$  and  $g$  are differentiable functions such that

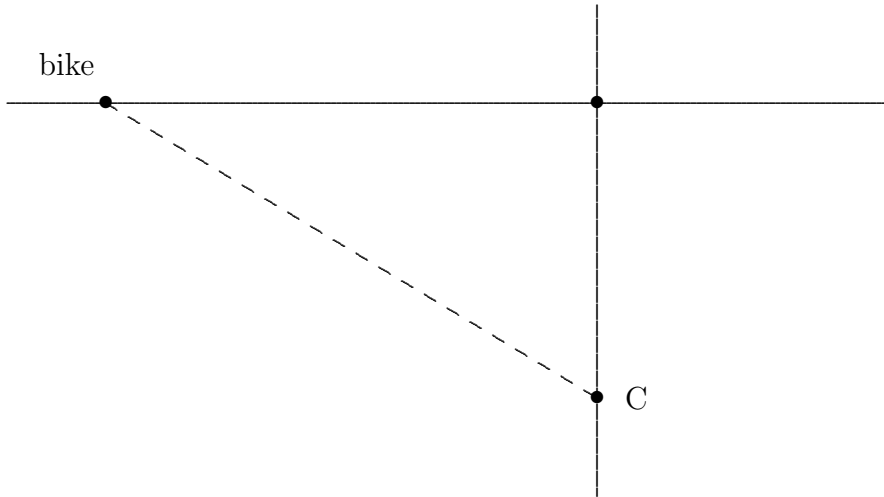
$$f(6) = 2, \quad f'(6) = 4, \quad g(2) = 3, \quad \text{and} \quad g'(2) = -7.$$

Find each of the following.

(a)  $h'(6)$  where  $h(x) = \arctan(f(x)^2)$ .

(b)  $k'(2)$  where  $k(x) = f\left(\frac{x^3}{4} \cdot g(x)\right)$

- (10) A cyclist starts 20 km west of an intersection and rides with a constant speed of 40 kilometers per hour toward the intersection. A cappuccino bar is located 5 kilometers south of the intersection at point  $C$ . At what rate is the distance between the cyclist and the cappuccino bar decreasing when the cyclist is halfway to the intersection?



Distance decreases at a rate of: \_\_\_\_\_