



**Record the correct answer to the following problems on the front page of this exam.**

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1. Suppose  $f(x) = x^{10} + 4x^5 + 2x^2 + 1$ . Find  $f''(1)$ .
  - (A) 91
  - (B) 110
  - (C) 134
  - (D) 174
  - (E) 230
  
2. Suppose  $y = 3x + 2$  is the equation of the line tangent to a function  $f$  at the point  $(1, f(1))$ . Find  $f(1)$  and  $f'(1)$ .
  - (A)  $f(1) = 5$  and  $f'(1) = 3$
  - (B)  $f(1) = 3$  and  $f'(1) = 5$
  - (C)  $f(1) = 3$  and  $f'(1) = 2$
  - (D)  $f(1) = 2$  and  $f'(1) = 3$
  - (E)  $f(1) = 5$  and  $f'(1) = 2$
  
3. Suppose  $f(x) = \tan^2(x)$ . Find  $f'(\frac{\pi}{4})$ .
  - (A) 4
  - (B) 2
  - (C) 1
  - (D) 0
  - (E)  $-2$

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4. Suppose  $f(x) = e^{2x} \cos(3x)$ . Find  $f'(\frac{\pi}{2})$ .

- (A)  $-e^\pi$
- (B)  $2e^\pi$
- (C)  $-2e^\pi$
- (D)  $-3e^\pi$
- (E)  $3e^\pi$

5. Suppose that the function  $y(x)$  satisfies the equation  $xy^2 + 2y = 2x$ . Find  $\frac{dy}{dx}$  at the point  $(2, 1)$ .

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{1}{4}$
- (D)  $\frac{1}{5}$
- (E)  $\frac{1}{6}$

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6. Suppose  $f = \sqrt{2 + \sqrt{x}}$ . Find  $f'(4)$ .

- (A)  $\frac{1}{16}$
- (B)  $\frac{1}{4}$
- (C)  $\frac{1}{2}$
- (D) 1
- (E) 2

7. Suppose  $f(x) = \frac{1 + x^2}{g(x) + x}$ , and  $g(0) = 2$  and  $g'(0) = 3$ . Find  $f'(0)$ .

- (A)  $-2$
- (B)  $-1$
- (C) 0
- (D) 1
- (E) 2

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8. Let  $g$  be the inverse function of the function  $f$ . Suppose  $f(0) = 2$ ,  $f'(0) = 3$ ,  $f(2) = 5$  and  $f'(2) = 4$ . Find  $g(2)$  and  $g'(2)$ .
- (A)  $g(2) = 0$  and  $g'(2) = 4$
  - (B)  $g(2) = 0$  and  $g'(2) = 1/4$
  - (C)  $g(2) = 0$  and  $g'(2) = 1/3$
  - (D)  $g(2) = 5$  and  $g'(2) = 1/4$
  - (E)  $g(2) = 5$  and  $g'(2) = 1/3$
9. Suppose that  $f(x) = e^x + 2e^{-x}$ . Find  $f^{(401)}(x)$ .
- (A)  $e^x + 2e^{-x}$
  - (B)  $e^x - 2e^{-x}$
  - (C)  $e^x + e^{-x}$
  - (D)  $e^x - e^{-x}$
  - (E) None of the above
10. Suppose the radius of a sphere is  $2t + 3$  meters ( $t$  is time and measured in seconds). Find the rate of change of the volume of the sphere measured in  $m^3/\text{sec}$  when  $t = 3$ . (The volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ .)
- (A)  $648\pi$
  - (B)  $728\pi$
  - (C)  $512\pi$
  - (D)  $484\pi$
  - (E) None of the above.

**Free Response Questions: Show your work!**

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11. (a) Let  $g(x) = \sec(e^{2x})$ . Find  $g'(x)$ .

(b) Find the coordinates of all points where the curve given by the equation

$$25x^2 + 16y^2 + 200x - 160y + 400 = 0$$

has horizontal tangent lines.

**Free Response Questions: Show your work!**

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12. For parts (a) and (b) below, assume that  $f$  and  $g$  are two functions such that  $f(0) = 1$ ,  $f'(0) = 0$ ,  $g(0) = 1$  and  $g'(0) = 2$ .

(a) Find  $h'(0)$  where  $h(x) = 3f(x)g(x) + [g(x)]^2$ .

(b) Find  $l'(0)$  where  $l(x) = \ln\left(\frac{f(x)}{g(x)}\right)$ .

**Free Response Questions: Show your work!**

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13. (a) Let  $f(x) = \sqrt{x+1}$ . Use the definition of the derivative as a limit to compute  $f'(0)$ . If you use any other method, you will not receive any points.

- (b) Find constants  $a$  and  $b$  such that the function

$$f(x) = \begin{cases} ax + b, & x \leq 1 \\ -(x-2)^2 + 4, & x > 1 \end{cases}$$

is differentiable at  $x = 1$ .



**Free Response Questions: Show your work!**

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14. A ladder of length 290 cm is leaning against a wall. The ladder begins to slide away from the wall. The bottom of the ladder moves away from the wall at a constant rate of 60 cm/sec. Find the rate of change of the height of the top of the ladder when the height of the top of the ladder is 200 cm.

**Free Response Questions: Show your work!**

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15. The position (measured in meters) of an object moving in a straight line is given by the equation

$$P(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 - 2t + 10, \quad t \geq 0.$$

- (a) Find the time when the velocity of the object is  $2 \text{ m/s}$ .

- (b) For what values of  $t$  is the acceleration of the object positive?