MA 113 Calculus I
Fall 2014
Tuesday, 21 October 2014

Name: $\qquad$

Section: $\qquad$

Last 4 digits of student ID \#: $\qquad$
This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

## On the multiple choice problems:

1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

## On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

| Question |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |

Exam Scores

| Question | Score | Total |
| :---: | ---: | ---: |
| MC |  | 50 |
| 11 |  | 10 |
| 12 |  | 10 |
| 13 |  | 10 |
| 14 |  | 10 |
| 15 |  | 10 |
| Total |  | 100 |

Record the correct answer to the following problems on the front page of this exam.

1. Suppose $f(x)=x^{10}+4 x^{5}+2 x^{2}+1$. Find $f^{\prime \prime}(1)$.
(A) 91
(B) 110
(C) 134
(D) 174
(E) 230
2. Suppose $y=3 x+2$ is the equation of the line tangent to a function $f$ at the point $(1, f(1))$. Find $f(1)$ and $f^{\prime}(1)$.
(A) $f(1)=5$ and $f^{\prime}(1)=3$
(B) $f(1)=3$ and $f^{\prime}(1)=5$
(C) $f(1)=3$ and $f^{\prime}(1)=2$
(D) $f(1)=2$ and $f^{\prime}(1)=3$
(E) $f(1)=5$ and $f^{\prime}(1)=2$
3. Suppose $f(x)=\tan ^{2}(x)$. Find $f^{\prime}\left(\frac{\pi}{4}\right)$.
(A) 4
(B) 2
(C) 1
(D) 0
(E) -2

Record the correct answer to the following problems on the front page of this exam.
4. Suppose $f(x)=e^{2 x} \cos (3 x)$. Find $f^{\prime}\left(\frac{\pi}{2}\right)$.
(A) $-e^{\pi}$
(B) $2 e^{\pi}$
(C) $-2 e^{\pi}$
(D) $-3 e^{\pi}$
(E) $3 e^{\pi}$
5. Suppose that the function $y(x)$ satisfies the equation $x y^{2}+2 y=2 x$. Find $\frac{d y}{d x}$ at the point $(2,1)$.
(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{1}{5}$
(E) $\frac{1}{6}$

Record the correct answer to the following problems on the front page of this exam.
6. Suppose $f=\sqrt{2+\sqrt{x}}$. Find $f^{\prime}(4)$.
(A) $\frac{1}{16}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) 2
7. Suppose $f(x)=\frac{1+x^{2}}{g(x)+x}$, and $g(0)=2$ and $g^{\prime}(0)=3$. Find $f^{\prime}(0)$.
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

## Record the correct answer to the following problems on the front page of this exam.

8. Let $g$ be the inverse function of the function $f$. Suppose $f(0)=2, f^{\prime}(0)=3, f(2)=5$ and $f^{\prime}(2)=4$. Find $g(2)$ and $g^{\prime}(2)$.
(A) $g(2)=0$ and $g^{\prime}(2)=4$
(B) $g(2)=0$ and $g^{\prime}(2)=1 / 4$
(C) $g(2)=0$ and $g^{\prime}(2)=1 / 3$
(D) $g(2)=5$ and $g^{\prime}(2)=1 / 4$
(E) $g(2)=5$ and $g^{\prime}(2)=1 / 3$
9. Suppose that $f(x)=e^{x}+2 e^{-x}$. Find $f^{(401)}(x)$.
(A) $e^{x}+2 e^{-x}$
(B) $e^{x}-2 e^{-x}$
(C) $e^{x}+e^{-x}$
(D) $e^{x}-e^{-x}$
(E) None of the above
10. Suppose the radius of a sphere is $2 t+3$ meters ( $t$ is time and measured in seconds). Find the rate of change of the volume of the sphere measured in $m^{3} / \mathrm{sec}$ when $t=3$. (The volume of a sphere of radius $r$ is $\frac{4}{3} \pi r^{3}$.)
(A) $648 \pi$
(B) $728 \pi$
(C) $512 \pi$
(D) $484 \pi$
(E) None of the above.
11. (a) Let $g(x)=\sec \left(e^{2 x}\right)$. Find $g^{\prime}(x)$.
(b) Find the coordinates of all points where the curve given by the equation

$$
25 x^{2}+16 y^{2}+200 x-160 y+400=0
$$

has horizontal tangent lines.
12. For parts (a) and (b) below, assume that $f$ and $g$ are two functions such that $f(0)=1, f^{\prime}(0)=0, g(0)=1$ and $g^{\prime}(0)=2$.
(a) Find $h^{\prime}(0)$ where $\left.h(x)=3 f(x) g(x)+[g(x))\right]^{2}$.
(b) Find $l^{\prime}(0)$ where $l(x)=\ln \left(\frac{f(x)}{g(x)}\right)$.
13. (a) Let $f(x)=\sqrt{x+1}$. Use the definition of the derivative as a limit to compute $f^{\prime}(0)$. If you use any other method, you will not receive any points.
(b) Find constants $a$ and $b$ such that the function

$$
f(x)= \begin{cases}a x+b, & x \leq 1 \\ -(x-2)^{2}+4, & x>1\end{cases}
$$

is differentiable at $x=1$.
14. A ladder of length 290 cm is leaning against a wall. The ladder begins to slide away from the wall. The bottom of the ladder moves away from the wall at a constant rate of $60 \mathrm{~cm} / \mathrm{sec}$. Find the rate of change of the height of the top of the ladder when the height of the top of the ladder is 200 cm .
15. The position (measured in meters) of an object moving in a straight line is given by the equation

$$
P(t)=\frac{1}{3} t^{3}-\frac{3}{2} t^{2}-2 t+10, t \geq 0 .
$$

(a) Find the time when the velocity of the object is $2 \mathrm{~m} / \mathrm{s}$.
(b) For what values of $t$ is the acceleration of the object positive?

