Solutions

MA 113 Calculus I — Fall 2016

Exam 2 — Tuesday, October 18, 2016

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

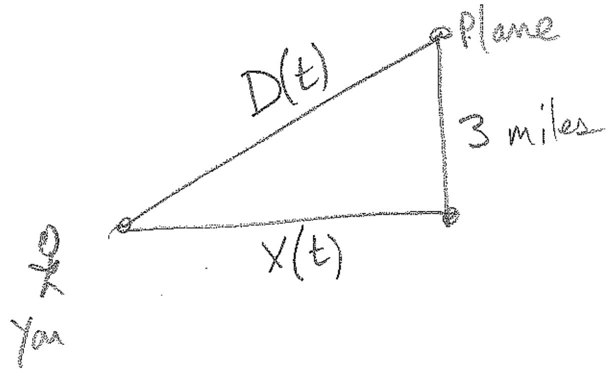
Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. A plane is flying directly away from you at 500 mph and at an altitude of 3 miles. The goal of this problem is to determine how fast the distance between you and the plane is increasing when the plane is flying over a point 4 miles from you.

(a) (4 points) Draw a diagram that shows the distance x along the ground from where you are to the point the plane is flying over, and the distance D from you to the plane. Be sure to label your position and that of the airplane.



(b) (6 points) Find a relationship between x and D and use the related rates technique to find how fast the distance between you and the plane is increasing when the plane is flying over a point 4 miles from you.

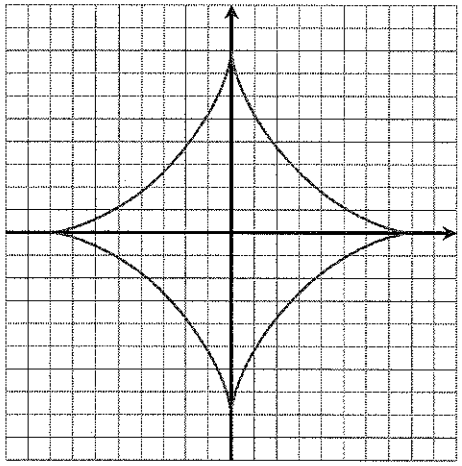
$$x^2 + 9 = D^2 \Rightarrow 2x \cdot \frac{dx}{dt} = 2D \cdot \frac{dD}{dt}.$$

When $x = 4$, $D = 5$; $\frac{dx}{dt} = 500 \text{ mph}$.

$$\Rightarrow \frac{dD}{dt} = 2x \frac{dx}{dt} \cdot \frac{1}{2D} = \frac{8}{10} \cdot 500 \text{ mph}$$

Free Response Questions: Show your work!

17. This question concerns the curve $x^{2/3} + y^{2/3} = 4$ graphed below.



(a) (4 points) Using implicit differentiation, find a formula for the slope dy/dx of the tangent line to the graph at the point (x, y).

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}}.$$

(b) (3 points) **Using your answer from part (a)**, find an equation involving x and y that gives a condition for the slope of the tangent line to the graph at (x, y) to be -1.

$$-\frac{x^{-1/3}}{y^{-1/3}} = -1.$$

(c) (3 points) **Using your answer from part(b)**, determine which of the points $(2\sqrt{2}, 2\sqrt{2})$, $(2\sqrt{2}, -2\sqrt{2})$, $(-2\sqrt{2}, 2\sqrt{2})$ and $(-2\sqrt{2}, -2\sqrt{2})$ have tangents with slope -1.

$(2\sqrt{2}, 2\sqrt{2})$ and $(-2\sqrt{2}, -2\sqrt{2})$.

Free Response Questions: Show your work!

18. Yahoo falls in McCreary County is said to be Kentucky's highest waterfall, at 113 feet. A stick rolls over the top with an initial velocity of -10 ft/sec. Gravity accelerates the stick downward at $g = 32$ ft/sec^2.

    (a) (4 points) Using Galileo's equation

    $$h(t) = h_0 + v_0 t - \frac{1}{2}gt^2$$

    find the function that gives the height of the stick off the ground in feet as a function of the time t in seconds.

    $h(t) = 113 + -10t - \frac{1}{2} \cdot 32t^2 \cdot$ ft.

    (b) (3 points) At what time t does the stick hit the ground? (Compute your answer to two decimal places, and be sure to state units!)

    $h(t) = 0 \Rightarrow 113 - 10t - 16t^2 = 0.$

    $t = \frac{10 \pm \sqrt{100 - 4 \cdot 113 \cdot -16}}{-32} \approx 2.36$

    (c) (3 points) What is the velocity of the stick when it hits the ground? (Compute your answer to two decimal places, and be sure to give units!)

    $v(t) = -10 - 32t$

    $\Rightarrow V(2.36) \approx -10 - 32(2.36) = -85.52$

Free Response Questions: Show your work!

19. This problem concerns the definition of the derivative using limits.

(a) (4 points) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$. *Hint*: Your definition should involve a limit.

See text.

(b) (6 points) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = 2/x$. An answer that is unsupported or uses differentiation rules will receive **no credit**.

$$\lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h \to 0} \frac{2x - 2(x+h)}{h \cdot x \cdot (x+h)} = \lim_{h \to 0} \frac{-2h}{h \cdot x(x+h)}$$

$$= \lim_{h \to 0} \frac{-2}{x(x+h)} = \frac{-2}{x^2}.$$