Calculus I
Examination 2

6 March 20(

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please: 1) check answers when possible, 2) clearly indicate your answ(and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*, 3) give exact answers, rather than decimal approximations to the answer.
Each question is followed by space to write your answer. Please write your solutions neatly in th space below the question. You are not expected to write each solution next to the statement of the question. If you use the back of a sheet, please indicate this by the question.
You are to answer two of the last three questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name Solutions

Section ________

Last four digits of student identification number ________

Question	Score	Total
p. 1/Q1		14
p. 2/Q2–3		14
p. 3/Q4–5		14
p. 4/Q6		14
p. 5/Q7–8		14
p. 6/Q9		14
p. 7/Q10		14
p. 8/Q11		14
Free	2	2
		100

14pt 1. Compute the derivatives of the following functions. Simplify your answers.

5pt (a) $f(x) = \sqrt{3+5x^2}$

5pt (b) $g(x) = \dfrac{x}{3+x^2}$

4pt (c) $h(x) = x^3 \sin(2x)$

(a) $f'(x) = \frac{1}{2}(3+5x^2)^{-\frac{1}{2}}(10x) = \dfrac{5x}{\sqrt{3+5x^2}}.$ 4pt 1pt

(b) $g'(x) = \dfrac{(3+x^2) - x(2x)}{(3+x^2)^2} = \dfrac{3-x^2}{(3+x^2)^2}$ or $\dfrac{(\sqrt{3}-x)(\sqrt{3}+x)}{(3+x^2)^2}.$ 4pt 1pt

(c) $h'(x) = 3x^2 \sin 2x + x^3 \cos(2x) \cdot 2.$

$= 3x^2 \sin 2x + 2x^3 \cos 2x$ 4pt

each mistake (−1) points.

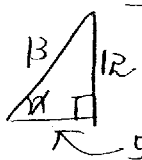
(a) $f'(x) = \underline{\dfrac{5x}{\sqrt{3+5x^2}}}$, (b) $g'(x) = \underline{\dfrac{3-x^2}{(3+x^2)^2}.}$,

(c) $h'(x) = \underline{3x^2 \sin 2x + 2x^3 \cos 2x}$

2. If $\sin(x) = \dfrac{12}{13}$ and x satisfies $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$. Find exact values for the following.

(a) $\cos(x)$, (b) $\tan(x)$, (c) $\cos(2x)$

3pt (a)



1pt

$\cos x = \dfrac{-5}{13}$ ← sign check 1pt. 1pt.

2pt (b) $\tan x = \dfrac{\sin x}{\cos x}$ 1pt $= \dfrac{12/13}{-5/13} = -\dfrac{12}{5}$ 1pt.

2pt (c) $\cos 2x = \cos^2 x - \sin^2 x$ 1pt $= \left(\dfrac{-5}{13}\right)^2 - \left(\dfrac{12}{13}\right)^2 = \dfrac{-119}{169}$ 1pt.

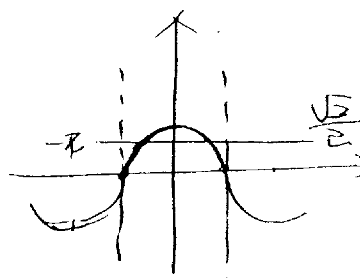
(a) $\cos(x) =$ $\underline{\quad -\dfrac{5}{13} \quad}$, (b) $\tan(x) =$ $\underline{\quad -\dfrac{12}{5} \quad}$,

(c) $\cos(2x) =$ $\underline{\quad \dfrac{-119}{169} \quad}$.

7pt 3. Find all values of x in the interval $(-\pi/2, \pi/2)$ where the tangent line to the graph of the function $f(x) = \sqrt{3}x - 2\sin(x)$ has a horizontal tangent line. Your answer(s) should be in radians and given as a multiple of π.

$f'(x) = \sqrt{3} - 2\cos x = 0$ 3pt.

$\cos x = \dfrac{\sqrt{3}}{2}$. 1pt.



$x = \dfrac{\pi}{6}$ and $-\dfrac{\pi}{6}$ 3pt

(1pt for each answer. & 1pt for realizing more than 1 answ

$x = \dfrac{\pi}{6}, -\dfrac{\pi}{6}$

4pt 4. (a) If $h(t) = \frac{1}{t^4}$, find the second derivative h''.

3pt (b) If $g(x) = \cos(x)$, find the 21st derivative $g^{(21)}$.

(a) $h'(t) = -4(t^{-5}) = \frac{-4}{t^5}$ 2pt

$h''(t) = 20t^{-6} = \frac{20}{t^6}$ 2pt.

(b) $g'(x) = -\sin x$

$g''(x) = -\cos x$

$g'''(x) = \sin x$

$g^{(4)}(x) = \cos x$

2pt (for recognizing repeating pattern)

$g^{(21)}(x) = g'(x) = -\sin x$ 1pt

(a) $h''(t) = \frac{20}{t^6}$, (b) $g^{(21)}(x) = -\sin x$

7pt 5. The radius of a circle is $r(t) = 3t^2 + 1$ centimeters at time t-minutes. The area of a circle radius r is $A = \pi r^2$. Give the rate of change of the area of the circle with respect to time when $t = 1$. Give an exact answer as a multiple of π.

$\frac{dA}{dt} = \pi r 2 \frac{dr}{dt}$ 3pt

$r(1) = 3 \cdot 1 + 1 = 4$ 1pt

$r'(t) = 6t$, 1pt

$r'(1) = 6$. 1pt

$\frac{dA}{dt} = \pi \cdot 2 \cdot 4 \cdot 6 = 48\pi$ 1pt.

48π

6. Consider the curve defined by $3x^2 - 4y^3 = 8$.

5pt (a) Use implicit differentiation to find $\frac{dy}{dx}$ at a point (x, y) on the curve $3x^2 - 4y^3 = 8$.

5pt (b) Find the equation of the tangent line to this curve at the point $(x, y) = (2, 1)$. Put your answer in the form $y = mx + b$.

4pt (c) Find the equation of the line that is perpendicular to the curve at the point $(x, y) = (2, 1)$. Put your answer in the form $y = mx + b$.

(a) $6x - 12y^2 \frac{dy}{dx} = 0$ 3pt (-1 for each mistake)

$\frac{dy}{dx} = \frac{-6x}{-12y^2} = \frac{x}{2y^2}$ 2pt

(b) $\left.\frac{dy}{dx}\right|_{at\,(x,y)=(2,1)} = \frac{2}{2\cdot 1} = 1$ 2pt ($f'(a)$ evaluation)

$y - 1 = 1(x-2)$ 2pt (1pt for $y - f(a) = f'(a)(x-a)$; 1pt for any form of line equ)

$y = x - 1$ 1pt

(c) slope $= -1$ 2pt

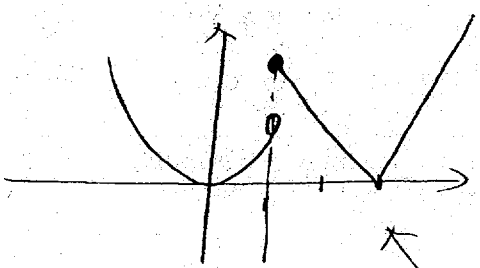
$y - 1 = -1(x-2)$ 1pt for any form of line

$y = -x + 3 \Leftarrow$ 1pt

(a) $\frac{dy}{dx} =$ $\frac{x}{2y^2}$, (b) $y = x - 1$,

(c) $y = -x + 3$

7pt 7. Let $f(x) = \begin{cases} x^2, & x < 1 \\ |x-3|, & x \geq 1 \end{cases}$. Find all critical numbers for f.



plot or idea
1pt.

$f'(x) = 2x, \quad x < 1$

$x = 0, \quad f'(0) = 0$. 2pt

$f'(x)$ D.N.E at $x = 3$. 2pt.

$f'(x)$ D.N.E at $x = 1$ 2pt

$x = 0, 3, 1$

7pt 8. Let $g(x) = x^3 - 12x$. Find the absolute minimum value and absolute maximum value for g on the interval $[-3, 5]$.

$g'(x) = 3x^2 - 12$.

$3x^2 - 12 = 0$ 1pt

$3(x^2 - 4) = 0$

$3(x-2)(x+2) = 0 \quad x = -2$ or 2. 1pt

1pt each

$g(2) = 8 - 12 \cdot 2 = -16$ min

$g(-2) = (-8) + 24 = 16$.

$g(-3) = -27 + 36 = 9$.

$g(5) = 125 - 60 = 65$ max

Finding min/max 1pt

Absolute maximum value <u>65</u>, Absolute minimum value <u>-16</u>

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

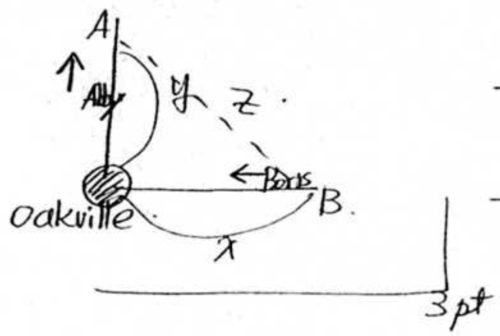
9. Abby is north of Oakville and driving north along Road A and Boris is east of Oakville and driving west along Road B. At 11:57 am, Boris is 5 kilometers east of Oakville and driving west at a speed of 60 kilometers/hour. At this same time, Abby is 10 kilometers north of Oakville and driving north at a speed of 50 kilometers/hour.

3pt (a) Make a sketch showing the location and direction of travel for Abby and Boris.

9pt (b) Find the rate of change of the distance between Abby and Boris at 11:57 am.

2pt (c) At 11:57 am, is the distance between Abby and Boris increasing, decreasing or not changing?

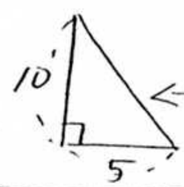
(a)



$\frac{dx}{dt} = -60$ km/h. $\qquad x = 5$

$\frac{dy}{dt} = 50$ km/h. $\qquad y = 10$

(b) $x^2 + y^2 = z^2$ 2pt



$10^2 + 5^2 = \sqrt{125}$

1pt

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ 2pt

$2 \cdot 5 \cdot (-60) + 2 \cdot 10 \cdot (50) = 2\sqrt{125} \cdot \frac{dz}{dt}$ 2pt

$-600 + 1000 = 2\sqrt{125}\,\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{\cancel{400}}{\cancel{2}\sqrt{125}} = \frac{200}{\sqrt{125}}$ km/h. 2pt.

(c) $\frac{dz}{dt}$: positive

increasing $\Leftarrow$ 2pt.

10. Consider the parabola $y = x^2$.

4pt (a) Write the equation of the tangent line to this parabola at a general point on the parabola (a, a^2).

10pt (b) Find all tangent lines to this parabola which pass through the point $(x, y) = (2, 3)$ Your solution must show how you found the tangent lines.

ⓐ $y - a^2 = 2a(x-a)$ | form 2pt

2pt.

ⓑ $3 - a^2 = 2a(2-a)$ | 3pt.

$3 - a^2 = 4a - 2a^2$

$a^2 - 4a + 3 = 0$

$(a-3)(a-1) = 0$

$a = 1$ or $3.$ } 3pt.

$y - 1 = 2(x-1)$, or $y - 9 = 6(x-3)$

$y = 2x - 1$ $\qquad$ $y = 6x - 9.$

2pt eac

11. (a) Give the definition of the linearization or linear approximation of a function f at a number a.
3pt

(b) The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. Find the linearization to the volume of a sphere at $r = 4$.
5pt

(c) If we cover a sphere of radius 4 centimeters with a layer of paint which is 0.01 centimeters thick, use the linear approximation in part (b) to estimate the volume of paint used.
6pt

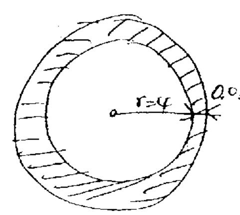
ⓐ $L(x) = f(a) + f'(a)(x-a)$ 3pt.

ⓑ $L(x) = f(4) + f'(4)(x-4)$ $f'(r) = \frac{4}{3}\pi 3r^2$ 2pt.

$= \frac{4}{3}\pi(4)^3 + \frac{4}{3}\cdot 3\pi(4)^2(x-4)$ 3pt.

$= 64\pi x - \frac{512}{3}\pi.$

ⓒ $r=4.$ $r-4=0.01.$



3pt

$dV = \frac{4}{3}3\pi(4)^2 dr.$

or.

$L(x) - f(4) = \frac{4}{3}.3\pi(4)^2(r-4)$

$dV = \frac{4}{\cancel{3}}\cdot\cancel{3}\pi(4)^2(0.01) \Leftarrow$ 2pt (inputting all the value

$= \underline{0.64\pi}. \Leftarrow$ 1pt.