

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

You are to answer *two of the last three questions*. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name: _____

Section: _____

Last four digits of student identification number: _____

Question	Score	Total
1		8
2		12
3		10
4		10
5		10
6		10
7		10
8		14
9		14
10		14
Free	2	2
		100

(1) Given that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$, use the limit laws to find the following limits.

(a) $\lim_{t \rightarrow 0} \frac{\sin(7t^2)}{3t}$.

(b) $\lim_{t \rightarrow 0^+} t \cot(3t)$.

(a) _____

(b) _____

(2) Find the following derivatives.

(a) $g'(t)$ at $t = 0$ when $g(t) = \frac{2t + 3}{t^2 + t + 4}$.

(b) $h'(x)$ when $h(x) = 3 \sin(4x^2) + 2 \tan(x)$.

(c) $f'(x)$ when $f(x) = (\sqrt{2 + x + x^4})^3$.

(a) $g'(0) =$ _____

(b) $h'(x) =$ _____

(c) $f'(x) =$ _____

(3) Find the following second derivatives:

(a) $g''(x)$ when $g(x) = (1 - 2x)^{-3}$.

(b) $h''(1)$ when $h(x) = (x - 2) \sin(\pi x)$.

(a) $g''(x) =$ _____

(b) $h''(1) =$ _____

(4) A particle is traveling along a straight line. Its position after t seconds is given by $s(t) = 2t^3 - 21t^2 + 72t + 10$ meters.

(a) Find the velocity and acceleration of the particle at time t .

(b) Determine all times, t , where the velocity is zero.

(a) (i) $v(t) =$ _____ (ii) $a(t) =$ _____

(b) _____

- (5) Consider the curve given by the equation $y^3 + yx^4 = 2$. Find the equation of the tangent line to this curve at $(1,1)$. Write your answer in the form $y = mx + b$.

Equation of tangent line: _____

- (6) The cost of making x campaign buttons for the upcoming presidential election is given by $C(x) = 2\sqrt{x+4}$ dollars.
- (a) Find a linear function $L(x)$ with $L(96) = C(96)$ and $L'(96) = C'(96)$.
- (b) Use (a) to find an approximation to $C(100)$ (as usual, show your work).

(a) $L(x) =$ _____

(b) $C(100) \approx$ _____

(7) Given the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, $n = 1, 2, \dots$ which is used in Newton's method.

(a) Briefly explain, using a diagram, how this formula is obtained. You do not need to actually derive the formula.

(b) Indicate how the above formula can be used to find the cube root of 9. In particular indicate (i) $f(x)$ and if $x_1 = 2$ find (ii) x_2 .

(b) (i) $f(x) =$ _____ (ii) $x_2 =$ _____

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

- (8) A baseball diamond is a square with 90 feet on a side. A batter hits the ball at home plate (one corner of the square) and runs toward first base with a speed of 24 feet per second (see diagram). At what rate is the runner's distance from second base decreasing when the runner is halfway to first base (you may give a decimal approximation as your answer)?

Rate of decrease = _____

(9) (a) State the chain rule. Use complete sentences.

(b) Illustrate the chain rule by computing the derivative of (i) $h(x) = f(x^5)$ and (ii) $k(x) = (f(x))^5$ at $x = 1$ when $f(1) = 2$ and $f'(1) = -1$.

(i) $h'(1) =$ _____ (ii) $k'(1) =$ _____

(10) Given the function, $f(x) = 3x^2 + 5x + 4$, whose graph is a parabola.

(a) Write the equation of the tangent line to this parabola at a point, $(a, 3a^2 + 5a + 4)$.

(b) Find all tangent lines to this parabola which pass through $(1,0)$. Your solution must show how you found the tangent lines.

(a) _____

(b) (i) _____ (ii) _____