MA 113 — Calculus I Spring 2009
Exam 2 March 10, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number on the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		8
2		12
3		10
4		8
5		9
6		10
7		8
8		16
9		16
10		16
Free	3	3
		100

(1) Given that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, use the limit laws to find the limits.

(a) $\lim_{t\to 0} e^t \frac{\sin(5t^3)}{2t}$;

(b) $\lim_{x\to 0} x \ln(x+e) \cdot \cot(4x)$.

(a) $\lim_{t\to 0} e^t \frac{\sin(5t^3)}{2t} =$ ______________

(b) $\lim_{x\to 0} x \ln(x+e) \cdot \cot(4x) =$ ______________

(2) Find the derivative of the following functions:

(a) $f(x) = \cos(4x) + 2\sqrt{x} + \ln x + \pi^3$;

(b) $g(x) = \dfrac{x^2}{x^4 + 2}$;

(c) $h(x) = x^3 \cdot e^{5x^2+1}$.

(a) $f'(x) =$ ____________________________________

(b) $g'(x) =$ ____________________________________

(c) $h'(x) =$ ____________________________________

(3) Use the differentiation rules to determine the following higher order derivatives. As always, show your work.

(a) Find $f''(x)$ if $f(x) = e^{x^2}$.

(b) Find $g''(5)$ if $g(x) = (x-4) \cdot \cos(\frac{\pi}{2}x)$.

(a) $f''(x) =$ ____________

(b) $g''(5) =$ ____________

(4) A particle is traveling along a straight line. Its position after t seconds is given by $s(t) = 2t^3 - 15t^2 + 36t + 1$ meters.

(a) Find the velocity and acceleration of the particle at time t.

(b) Determine the total distance traveled by the particle during the first four seconds.

(a) $v(t) =$ ____________ m/s, $a(t) =$ ____________ m/s^2

(b) Total distance is ____________ meters.

(5) Consider the curve described given by the equation $(3-x)\cdot y^2 = x^2$. Find the equation of the line tangent to this curve at the point $(2,2)$. Write your answer in the form $y = mx+b$. As always, show your work.

Equation of the tangent line is $y =$ ____________

(6) As usual, show your work in answering the following questions.

(a) Find $f'(1)$ if $f(x) = \ln\left(\dfrac{x+1}{x^2+1}\right)$.

(b) Find $g'(\pi)$ if $g(x) = (e + \sin x)^{\sin x}$.

(a) $f'(1) =$ ______________

(b) $g'(\pi) =$ ______________

(7) A radioactive substance known to be 500 years old is 80% decayed. What is the half-life of the substance correct to two decimal places, this is, after how many years was half of the original amount m_0 decayed? (Use the fact that the mass of a radioactive substance changes over time according to the formula $m(t) = m_0 e^{kt}$, where k is a constant.)

The half-life is ____________ years.

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) Consider the function $f(x) = x^2 - 2x + 2$ whose graph is a parabola.

(a) Write the equation of the tangent line to this parabola at a point $(a, f(a))$.

(b) Find the equation of each tangent line to this parabola that passes through the point $(3, 4)$. As always, show your work.

(a) ______________________________

(b) $y =$ ____________________ $y =$ ____________________

(9) (a) State the chain rule. Use complete sentences.

(b) Suppose f and g are differentiable functions such that $f(2) = 3, f'(2) = -1, g(2) = 1/4$, and $g'(2) = 2$. Find each of the following:

(i) $h'(2)$ where $h(x) = \ln([f(x)]^2)$;

(ii) $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.

(i) $h'(2) =$ ____________ (ii) $l'(2) =$ ____________

(10) A small boat is being pulled into a dock that is 2 feet above the water surface by a rope attached to the bow and going up over a pulley located the end of the dock. The rope is attached to the boat at a point that is 2 feet above the surface of the water and the pulley is 5 feet above the dock. The rope is being reeled in at the rate of 4 feet per second. How fast is the boat coming towards the dock at the moment the bow is 12 feet away from the base of the pulley stand? (Recall that $12^2 + 5^2 = 13^2$.)

The velocity is ____________ ft/s.