MA 113 Calculus I — Spring 2015
Exam 2 — Tuesday, 10 March 2015

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

E, B, D, C, A B, B, C, D, D

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Suppose $f(x) = e^{2x}(x^3 + 4x)$. Find $f'(1)$.

   (A) $13e^2$
   (B) $14e^2$
   (C) $15e^2$
   (D) $16e^2$
   (E) $17e^2$

2. Suppose $f(x) = x^4 - x^3 + 7x + 11$. Find the equation of the tangent line to the graph of $f(x)$ at the point $x = 1$.

   (A) $y = 8x - 26$
   (B) $y = 8x + 10$
   (C) $y = -8x + 10$
   (D) $y = -8x - 18$
   (E) $y = -8x + 12$

3. Suppose $f(x) = \dfrac{x^2 + 3x + 4}{x - 7}$. Find $f'(2)$.

   (A) $\dfrac{-46}{25}$
   (B) $\dfrac{-47}{25}$
   (C) $\dfrac{-48}{25}$
   (D) $\dfrac{-49}{25}$
   (E) -2

Record the correct answer to the following problems on the front page of this exam.

4. Suppose $f(x) = \ln(x^4 + 8x^2 + 10)$. Find $f'(1)$.

   (A) $\dfrac{18}{19}$

   (B) 1

   (C) $\dfrac{20}{19}$

   (D) $\dfrac{21}{19}$

   (E) $\dfrac{22}{19}$

5. Suppose that the function $y(x)$ satisfies the equation $x^2y^3 + 4y = 5x$. Find $\dfrac{dy}{dx}$ at the point $(1, 1)$.

   (A) $\dfrac{3}{7}$

   (B) $\dfrac{4}{7}$

   (C) $\dfrac{5}{7}$

   (D) $\dfrac{6}{7}$

   (E) 1

Record the correct answer to the following problems on the front page of this exam.

6. Suppose $f(x) = x^2 e^{-2x}$. Find $f''(1)$.

   (A) $-e^{-2}$

   (B) $-2e^{-2}$

   (C) $-3e^{-2}$

   (D) $-4e^{-2}$

   (E) $-5e^{-2}$

7. Suppose $f(x) = \tan(x)\sec(x)$. Find $f'(\pi)$.

   (A) -2

   (B) -1

   (C) 0

   (D) 1

   (E) 2

Record the correct answer to the following problems on the front page of this exam.

8. Let $g(x)$ be the inverse function of the function $f(x)$. Suppose $f(5) = 7$, $f'(5) = 3$, $f(7) = 4$ and $f'(7) = 2$. Find $g(7)$ and $g'(7)$.

   (A) $g(7) = 2$ and $g'(7) = 1/2$

   (B) $g(7) = 4$ and $g'(7) = 1/2$

   (C) $g(7) = 5$ and $g'(7) = 1/3$

   (D) $g(7) = 5$ and $g'(7) = 1/2$

   (E) $g(7) = 2$ and $g'(7) = 1/3$

9. Suppose that $f(x) = \sqrt{2x^2 + 6x + 12}$. Find $f\,'(1)$.

   (A) $4\sqrt{20}$

   (B) $2\sqrt{20}$

   (C) $\dfrac{4}{\sqrt{20}}$

   (D) $\dfrac{\sqrt{5}}{2}$

   (E) None of the above

10. The radius of a sphere is $2t^2 + t$ meters where t is the time measured in seconds. Find the rate of change of the volume of the sphere measured in m^3/sec when $t = 1$. (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

    (A) 144π

    (B) 150π

    (C) 162π

    (D) 180π

    (E) None of the above.

Free Response Questions: Show your work!

11. (a) Let $g(x) = \ln(1 + e^{2x})$. Find the equation of the tangent line to the graph of $g(x)$ at $x = 0$.

$g'(x) = \frac{2e^{2x}}{1+e^{2x}}$, so $g'(0) = \frac{2}{1+1} = 1$. Since $g(0) = \ln(2)$, the equation of the tangent line is $y - \ln(2) = 1 \cdot (x - 0)$, which is just $y = x + \ln(2)$.

(b) Find the slope of the tangent line to the graph of the curve given by the equation $2x^2 + 3y^2 + 10y = 15$ at the point $(1, 1)$.

Differentiate to obtain $4x + 6yy' + 10y' = 0$. Then $y' = \dfrac{-4x}{6y + 10}$. Thus y' evaluated at $(1, 1)$ is $\frac{-4}{16} = \frac{-1}{4}$, which is the slope of the tangent line to the graph at $(1, 1)$.

12. For parts (a) and (b) below, assume that f and g are two functions such that $f(0) = 1$, $f'(0) = 0$, $g(0) = 1$ and $g'(0) = 2$.

    (a) Find $h'(0)$ where $h(x) = \dfrac{f(x)}{f(x) + g(x)}$.

    $h'(x) = \dfrac{(f+g)f' - f(f'+g')}{(f+g)^2}$. Then $h'(0) = \dfrac{2 \cdot 0 - 1 \cdot 2}{2^2} = \dfrac{-1}{2}$.

    (b) Find $l'(0)$ where $l(x) = e^{f(x)-g(x)}$.

    $l' = e^{f-g}(f' - g')$. Then $l'(0) = e^0(0 - 2) = -2$.

Free Response Questions: Show your work!

13. (a) State the definition of the derivative of a function $f(x)$.

The derivative $f'(x)$ is defined by $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

(b) Let $f(x) = \dfrac{1}{x+2}$. Use the definition of the derivative as a limit to compute $f'(0)$. If you use any other method, you will not receive any points.

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$
$$= \lim_{h\to 0} \frac{2-(2+h)}{h\cdot 2(2+h)} = \lim_{h\to 0} \frac{-h}{h\cdot 2(2+h)} = \lim_{h\to 0} \frac{-1}{2(2+h)} = \frac{-1}{4}.$$

Free Response Questions: Show your work!

14. An object is thrown upward with an initial velocity of 40.4 meters per second from a height of 100 meters. Assume that gravity is the only force acting on this object ($g = 9.8\, m/s^2$). (Use Galileo's equation $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$.)

    (a) When does the object reach its maximum height?

    $s(t) = 100 + 40.4t - 4.9t^2$, $v(t) = 40.4 - 9.8t$. The maximum height occurs when $v(t) = 0$. Thus $40.4 - 9.8t = 0$, so $t = 40.4/9.8$.

    (b) Find the height of the object when its velocity is $11 m/s$.

    We set $v(t) = 11$. Then $40.4 - 9.8t = 11$, $9.8t = 29.4$, $t = 3$. The desired height is $s(3) = 100 + 40.4(3) - 4.9(3^2) = 177.1$.

Free Response Questions: Show your work!

15. (a) Suppose $f(x) = \sin(\sqrt{1 + e^{2x}})$. Find $f'(x)$.

$f'(x) = \cos(\sqrt{1 + e^{2x}}) \cdot \frac{1}{2}(1 + e^{2x})^{\frac{-1}{2}}(2e^{2x})$.

(b) Find $\lim_{h \to 0} \dfrac{\sec(\frac{\pi}{4} + h) - \sec(\frac{\pi}{4})}{h}$ by interpreting this limit as a derivative and computing the derivative.

The limit is given by $\sec'(\frac{\pi}{4}) = \sec(\frac{\pi}{4})\tan(\frac{\pi}{4}) = \sqrt{2} \cdot 1 = \sqrt{2}$.