Exam 2

Form A

Name: ______________________ Section and/or TA: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

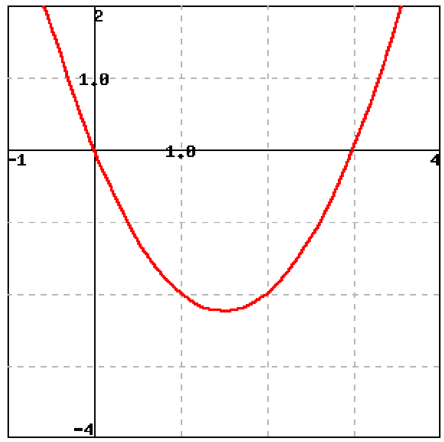
Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Calculate the slope of the secant line through the points on the graph where $x = 1$ and $x = 3$.



A. -1

B. 2

C. 0

D. 1

E. -2

2. Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 3x - 2$.

A. $f(3) = 2, f'(3) = 3$

B. $f(3) = 7, f'(3) = 3$

C. $f(3) = -2, f'(3) = 3$

D. $f(3) = 3, f'(3) = 2$

E. $f(3) = 9, f'(3) = 2$

3. Determine coefficients a and b such that $p(x) = x^2 + ax + b$ satisfies $p(1) = 11$ and $p'(1) = 11$.

   A. $a = 1, b = 9$

   B. $a = 10, b = 0$

   C. $a = 0, b = 10$

   D. $a = 8, b = 3$

   E. $a = 9, b = 1$

4. The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

   A. 190 cm^2/sec

   B. 224 cm^2/sec

   C. 140 cm^2/sec

   D. 24 cm^2/sec

   E. 200 cm^2/sec

5. Find a formula for $\frac{dy}{dx}$ in terms of x and y, where $x^4y + 4xy^4 = x + y$.

   A. $\frac{dy}{dx} = \frac{1 - x^4y - 16xy^3}{4x^3y + 4y^4 - 1}$

   B. $\frac{dy}{dx} = \frac{-1}{1 - 4x^3 - 16y^3}$

   C. $\frac{dy}{dx} = 4x^3y + x^4 + 4y^4 + 4x - 1$

   D. $\frac{dy}{dx} = \frac{4x^3y + 4y^4 - 1}{1 - x^4 - 16xy^3}$

   E. $\frac{dy}{dx} = \frac{-1}{1 - x^4 - 16x}$

6. Find $f'''(x)$ where $f(x) = xe^x$.

   A. $f'''(x) = e^x$

   B. $f'''(x) = (x+3)e^x$

   C. $f'''(x) = (x+1)e^x$

   D. $f'''(x) = (x+2)e^x$

   E. $f'''(x) = 3xe^x$

7. Find $f'(x)$ in terms of $g'(x)$ where $f(x) = x^2[g(x)]^2$.

   A. $f'(x) = 2x[g(x)]^2 + 2x^2 g(x)g'(x)$

   B. $f'(x) = 4xg'(x)$

   C. $f'(x) = 2x[g'(x)]^2$

   D. $f'(x) = 2x[g(x)]^2 + x^2[g'(x)]^2$

   E. $f'(x) = 4xg(x)g'(x)$

8. Find the derivative of $g(x) = x \arctan(x)$. (Remember that $\arctan(x)$ is the same as $\tan^{-1}(x)$.)

   A. $g'(x) = \dfrac{1}{1+x^2}$

   B. $g'(x) = \dfrac{x}{1+x^2}$

   C. $g'(x) = \arctan(x) + \dfrac{x}{1+x^2}$

   D. $g'(x) = \arctan(x) + \dfrac{1}{1+x^2}$

   E. $g'(x) = \arctan(x)$

9. Find the derivative of

$$h(x) = \frac{\ln(x^2)}{x^5}.$$

A. $h'(x) = \dfrac{1}{5x^6}$

B. $h'(x) = \dfrac{2}{5x^5}$

C. $h'(x) = \dfrac{1 - 5x\ln(x^2)}{x^7}$

D. $h'(x) = \dfrac{2 - 5\ln(x^2)}{x^6}$

E. $h'(x) = \dfrac{5\ln(x^2) - 2}{x^6}$

10. Differentiate

$$f(x) = \frac{\cos(2x)}{1 - x^2}$$

A. $f'(x) = \dfrac{\sin(2x)}{2x}$

B. $f'(x) = \dfrac{-2x\cos(2x) + 2(1 - x^2)\sin(2x)}{(1 - x^2)^2}$

C. $f'(x) = \dfrac{2(1 - x^2)\sin(2x) + 2x\cos(2x)}{(1 - x^2)^2}$

D. $f'(x) = \dfrac{-2(1 - x^2)\sin(2x) + 2x\cos(2x)}{1 - x^4}$

E. $f'(x) = \dfrac{-2(1 - x^2)\sin(2x) + 2x\cos(2x)}{(1 - x^2)^2}$

11. Suppose that $g(x) = \sin(x^2 - x - 6)$.
Find $g'(3)$

A. $\cos(5)$

B. 1

C. 0

D. 5

E. $\sin(5)$

12. The displacement (in meters) of a particle moving in a straight line is given by $s = 2t^2 - 6t + 5$, where t is measured in seconds. Find the average velocity over the time interval $[6, 8]$.

A. 63 m/sec

B. 4 m/sec

C. 44 m/sec

D. 8 m/sec

E. 22 m/sec

Free Response Questions
Show all of your work

13. Find the derivatives of the following functions.

(a) $f(x) = \ln(\tan(x))$.

(b) $g(x) = \dfrac{4}{x^3} - \dfrac{6}{x^2} - \dfrac{8}{x} + 10$.

(c) $h(x) = 4\ln(x^2 e^{x^2})$.

(d) $j(x) = \arcsin(2x)$

14. (a) Find the equation of the tangent line to $y^2 = 5x^4 - x^2$ at the point $(1, 2)$.

(b) Find $\lim_{x \to 0} \frac{\sin(3x)}{7x}$. (You may **NOT** use L'Hôpital's Rule to evaluate this.)

15. Let $f(x) = \dfrac{x^3}{x+8}$.

(a) Find the derivative $f'(x)$.

(b) Find the equation of the tangent line to $f(x)$ at the point where $x = 2$.

16. This problem concerns the definition of the derivative using limits.

    (a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$. *Hint*: Your definition should involve a limit.

    (b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = x^2 + x - 1$. An answer that is unsupported or uses differentiation rules will receive **no credit**.