Exam 2

Form A

Name: _____ Section and/or TA: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 14 multiple choice questions that count 5 points each and 3 free response questions that count 10 points each. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**8**	A	B	C	D	E
2	A	B	C	D	E	**9**	A	B	C	D	E
3	A	B	C	D	E	**10**	A	B	C	D	E
4	A	B	C	D	E	**11**	A	B	C	D	E
5	A	B	C	D	E	**12**	A	B	C	D	E
6	A	B	C	D	E	**13**	A	B	C	D	E
7	A	B	C	D	E	**14**	A	B	C	D	E

SCORE

Multiple Choice	15	16	17	Total Score
70	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Find the derivative of $\sin(x^3 + x)$.

   A. $\cos(x^3 + x)$

   B. $\sin(3x^2 + 1)$

   C. $\cos(3x^2 + 1)$

   D. $(3x^2 + 1)\cos(x^3 + x)$

   E. $(3x^2 + 1)\sin(x^3 + x)$

2. Suppose $f(3) = 5$ and $f'(3) = -3$ and let $g(x) = 2xf(x)$. Find $g'(3)$

   A. -18

   B. -15

   C. -8

   D. -6

   E. 28

3. Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 3x - 2$.

   A. $f(3) = 3,\ f'(3) = -2$

   B. $f(3) = -2,\ f'(3) = 3$

   C. $f(3) = 2,\ f'(3) = 3$

   D. $f(3) = 7,\ f'(3) = 3$

   E. $f(3) = 11,\ f'(3) = 3$

4. Find a formula for $\dfrac{dy}{dx}$ in terms of x and y where $x^2y - xy^2 = 2$.

   A. $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2}$

   B. $\dfrac{dy}{dx} = -\dfrac{y^2 + 2xy}{x^2 + 2xy}$

   C. $\dfrac{dy}{dx} = \dfrac{y^2 - 2x}{x^2 - 2y}$

   D. $\dfrac{dy}{dx} = \dfrac{x^2}{2xy}$

   E. $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2xy}$

5. Find the slope of the tangent line to the graph of $f(x) = x^3 \ln(x^2)$ at $x = e$.

A. $8e^2$

B. $6e^2 + 2e$

C. $6e^2 + e$

D. $6e$

E. 3

6. Find an equation of the tangent line to $4x^2 + 9y^2 = 72$ at the point $(3, -2)$.

A. $y = -\frac{2}{3}x$

B. $y = \frac{3}{2}x - \frac{13}{2}$

C. $y = \frac{2}{3}x + \frac{13}{3}$

D. $y = \frac{2}{3}x - 4$

E. $y = \frac{2}{3}x$

7. Let $h(x) = \dfrac{x^3+1}{x^2+1}$. Find $h'(1)$.

   A. -2

   B. $1/2$

   C. 1

   D. $3/2$

   E. 2

8. A cantaloupe is dropped off a tall building so that its height in meters at time t in seconds is $h(t) = -4.9t^2 + 98$. Find the velocity when it hits the ground. Give your answer correctly rounded to one decimal place.

   A. -43.4

   B. -43.6

   C. -43.8

   D. -44.2

   E. -44.4

9. Find the derivative of $g(x) = \tan(3x) + \sin(x^2)$.

   A. $g'(x) = \sec^2(3x) + \cos(x^2)$

   B. $g'(x) = 3\sec^2(3x) + 2x\cos(x^2)$

   C. $g'(x) = \sec^2(3x) + \cos(2x)$

   D. $g'(x) = 3\tan(3x) + 2x\cos(x^2)$

   E. $g'(x) = 3\tan(3x) + \cos(2x)$

10. Chromium-51 has a half-life of 28 days. A sample has a mass of 50 mg initially. Find the mass remaining after 30 days rounded to two decimal places.

    A. 8.70 mg

    B. 22.75 mg

    C. 23.79 mg

    D. 24.81 mg

    E. 25.00 mg

11. Find $f'(x)$ in terms of $g(x)$ and $g'(x)$ where $f(x) = [g(x)]^3$.

A. $f'(x) = 3g'(x)$

B. $f'(x) = 3[g(x)]^2$

C. $f'(x) = 3[g'(x)]^2$

D. $f'(x) = 3[g(x)]^2 g'(x)$

E. $f'(x) = 3[g(x)]^2(xg'(x) + g(x))$

12. Differentiate

$$f(x) = \frac{x^5}{1 - x^4}.$$

A. $f'(x) = \dfrac{5x^4}{1 - 4x^3}$

B. $f'(x) = \dfrac{(1 - x^4)^2}{x^4(5x - 4)}$

C. $f'(x) = \dfrac{x^4(5x - 4)}{(1 - x^4)^2}$

D. $f'(x) = \dfrac{x^4(1 - x^4)}{(5 - x^4)^2}$

E. $f'(x) = \dfrac{x^4(5 - x^4)}{(1 - x^4)^2}$

13. Find the derivative of

$$g(x) = x^5 \ln(9x).$$

A. $g'(x) = x^4(1 + 5\ln(9x))$

B. $g'(x) = x^4\left(\frac{1}{9} + 5\ln(9x)\right)$

C. $g'(x) = 1 + \frac{\ln(9x)}{9x}$

D. $g'(x) = \frac{5}{9}x^3$

E. $g'(x) = x^4(5\ln(9x) - 1)$

14. Let $f(x) = e^{(x^2)}$. Find $f''(x)$.

A. $(2 + 2x^2)e^{(x^2)}$

B. $(2 + 4x^2)e^{(x^2)}$

C. $(2 + 2x)e^{(x^2)}$

D. $(2 + 4x)e^{(x^2)}$

E. $2xe^{(x^2)}$

Free Response Questions
Show all of your work

15. (a) Find dy/dx for the curve $y^2 - 5xy + 6x^2 = 2$.

(b) Find the slope of the tangent line to $y^2 - 5xy + 6x^2 = 2$ at the point $(1, 4)$.

(c) Find an equation of the tangent line to $y^2 - 5xy + 6x^2 = 2$ at the point $(1, 4)$.

16. Find the derivatives of the following functions

(a) $f(x) = \ln(\sec(x))$

(b) $g(x) = x^2 e^{3x}$

(c) $h(x) = \dfrac{\sin(x)}{x^4}$

(d) $j(x) = \arctan(x^2)$

(e) $k(x) = \dfrac{2}{x^2} - \dfrac{3}{x} + 5 + 4x^2 + x^9$

17. The length of a rectangle is increasing at a rate of 12 cm/sec and its width is increasing at a rate of 4 cm/sec.

    (a) Find an equation that relates the area (A) of the rectangle to its length (L) and its width (W).

    (b) Find an equation that relates the rate of change of the area of the rectangle, dA/dt, to the rates of change of the length and the width, dL/dt and dW/dt.

    (c) If the length is 25 cm and the width is 15 cm, how fast is the area of the rectangle increasing?