MA 113 - Calculus I Spring 2002
THIRD MIDTERM 04/09/2002
Name: $\qquad$ Sec.: $\qquad$

| SEC. | INSTRUCTORS | T.A.'S | LECTURES | RECITATIONS |
| :--- | :--- | :--- | :--- | :--- |
| 001 | A. Corso | B. Bennewitz | MWF 8:00-8:50, CB 204 | TR 8:00-9:15, CB 341 |
| 002 | A. Corso | B. Bennewitz | MWF 8:00-8:50, CB 204 | TR 9:30-10:45, CB 345 |
| 004 | M. Silhavy | H. Song | MWF 10:00-10:50, CB 214 | TR 8:00-9:15, CB 349 |
| 005 | M. Silhavy | C. Budovsky | MWF 10:00-10:50, CB 214 | TR 2:00-3:15, CB 343 |
| 006 | M. Silhavy | H. Song | MWF 10:00-10:50, CB 214 | TR 3:30-4:45, CB 345 |
| 007 | A. Martin | M. Neu | MWF 12:00-12:50, CB 208 | TR 9:30-10:45, CB 347 |
| 008 | A. Martin | Y. Jia | MWF 12:00-12:50, CB 208 | TR 11:00-12:15, CB 347 |
| 009 | A. Martin | Y. Jia | MWF 12:00-12:50, CB 208 | TR 12:30-1:45, CB 349 |
| 010 | M. Silhavy | C. Budovsky | MWF 2:00-2:50, CB 204 | TR 12:30-1:45, CB 345 |
| 011 | M. Silhavy | M. Slone | MWF 2:00-2:50, CB 204 | TR 2:00-3:15, CB 345 |
| 012 | M. Silhavy | M. Slone | MWF 2:00-2:50, CB 204 | TR 3:30-4:45, CB 349 |

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may receive NO credit).

| QUESTION | SCORE | TOTAL |
| :---: | :---: | :---: |
| $\mathbf{1 .}$ |  | 10 |
| $\mathbf{2 .}$ |  | 10 |
| $\mathbf{3 .}$ |  | 15 |
| $\mathbf{4 .}$ |  | 8 |
| $\mathbf{5 .}$ |  | 10 |
| $\mathbf{6 .}$ |  | 15 |
| $\mathbf{7 .}$ |  | 10 |
| $\mathbf{8 .}$ |  | 10 |
| $\mathbf{9 .}$ |  | 12 |
| TOTAL |  | 100 |

1. Find all the critical values and the absolute maximum and absolute minimum values for

$$
f(x)=3 x^{4}-16 x^{3}+18 x^{2}
$$

on the closed interval $\quad-1 \leq x \leq 4$.
pts: /10
2. (a) Does the Mean Value Theorem apply to the function $f(x)=\frac{x+1}{x-1}$ on the interval $2 \leq x \leq 3$ ? Why? If so, find all possible values of $c$ for which the Mean Value Theorem holds on the given interval.
(b) Same as (a), but on the new interval $0.5 \leq x \leq 1.5$.
pts: /10

## 3. Consider the function: <br> $$
f(x)=x^{4}\left(x^{2}-3\right) .
$$

Each question is worth 5 points.
(a) Determine the intervals where the graph of $f(x)$ is increasing or decreasing. Find the values of $f(x)$ at the local maxima and minima of $f(x)$.
(b) Determine the intervals where the graph of $f(x)$ is concave up or down.

Find the values of $f(x)$ at the inflection points of $f(x)$.
(c) Sketch the graph of $f(x)$.

Make sure to label the local maxima, the local minima and the inflection points of $f(x)$.
4. Without using a calculator, show that the equation

$$
x^{101}+x^{51}+x-1=0
$$

has exactly one real root.
pts: /8
5. Show that if $x>0$ then $\quad x+\frac{4}{x^{2}} \geq 3$.
pts: /10
6. Each question is worth 5 points.
(a) $\lim _{x \rightarrow \infty} \frac{\sqrt{x}+3}{3-2 x}=$
(b) $\lim _{x \rightarrow \infty} \frac{2 \sqrt{1+9 x^{2}}}{9-16 x}=$
(c) Find the vertical and horizontal asymptotes of the curve

$$
f(x)=\frac{3 x^{2}+4}{2-x^{2}}
$$

Compute $\lim _{x \longrightarrow a^{+}} f(x)$ and $\lim _{x \longrightarrow a^{-}} f(x)$ for all the values of ' $a$ ' such that the line $x=a$ is a vertical asymptote of the given function $f(x)$.
7. Each problem is worth 5 points.
(a) The graph of a function $f(x)$ is shown. Which graph is an antiderivative of $f(x)$ and why?

(b) Find the most general antiderivative of: $\quad f(x)=x^{3}+\sqrt{x}-2 \cos (2 x)$.
pts: $/ 10$
8. A swimmer $S$ is in the ocean 100 meters from a straight shoreline. A person $P$ in distress is located on the shoreline 300 meters from the point on the shoreline closest to the swimmer.


If the swimmer can swim 3 meters per second and run 5 meters per second, what path should the swimmer follow in order to reach the person in distress as quickly as possible?
9. The graph of the derivative $f^{\prime}(x)$ of a function $f(x)$ is shown:


Each question is worth 3 points.
(a) On what intervals is $f(x)$ increasing or decreasing?
(b) At what values of $x$ does $f(x)$ attains a local maximum or minimum?
(c) On what intervals is $f(x)$ concave up or down?
(d) State the $x$-coordinates of the inflection points.

