MA 113 — Calculus I Fall 2009
Exam 3 November 17, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		9
2		8
3		13
4		8
5		10
6		8
7		10
8		15
9		15
10		15
Free	4	4
		100

(1) (9 points) Consider the function

$$g(x) = \frac{x}{x^2+1}$$

which has derivatives

$$g'(x) = \frac{(1-x^2)}{(x^2+1)^2}$$

and

$$g''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}.$$

Give exact answers to the following questions!

(a) (6 points) Find the intervals where the graph of g is concave up and concave down.

(b) (3 points) Find the inflection points of g (x- and y-coordinates).

(a) Interval(s) where the graph of g is

concave up: ____________________________

concave down: ____________________________

(b) Inflection point(s): ____________________________

(2) (8 points) Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 6x^2 + 9x + 2$$

on the interval $[-1, 4]$.

(a) Absolute maximum value ________________ at $x =$ ________________

(b) Absolute minimum value ________________ at $x =$ ________________

(3) (13 points) Consider the function $f(x) = \ln(x^2 - 6x + 11)$.
Give exact answers to the following questions!

(a) (4 points) Find the domain of f.

(b) (3 points) Find the critical number(s) of f.

(c) (3 points) Find the intervals of increase and decrease for f.

(d) (3 points) Find the local extremum or extrema of f. Be sure to give both the x-coordinate and the value of $f(x)$. Indicate whether each is a local maximum or a local minimum.

(a) Domain: ____________________

(b) Critical number(s): ____________________

(c) Interval(s) of increase: ____________________ and decrease: ____________________

(d) Local extremum/extrema: ____

(4) (8 points) Let $g(x) = \sqrt{25 + x}$. Find the linear approximation to g at $x = 0$. Use the linear approximation to estimate $\sqrt{25.06}$.

Linear approximation ____________________, $\sqrt{25.06} \approx$ ____________________

(5) (10 points) Find the general antiderivative of each of the following functions.

(a) (3 points) $f(t) = 3\sec^2(t) + \cos(t)$

(b) (3 points) $g(x) = x^2 - 2x + 4e^x$

(c) (4 points) $h(w) = w^{3/2} + 1/w$

(a) General antiderivative of $f(t)$ is ______________________________

(b) General antiderivative of $g(x)$ is ______________________________

(c) General antiderivative of $h(w)$ is ______________________________

(6) (8 points) A particle is moving with acceleration $a(t) = 10\sin(t) + 3\cos(t)$ meters/sec^2. Its initial position is $s(0) = 0$ meters and its initial velocity is $v(0) = 5$ meters per second.

(a) (4 points) Find $v(t)$, the velocity of the particle, at any time t. Be sure to specify the units.

(b) (4 points) Find $s(t)$, the position of the particle, at any time t. Be sure to specify the units.

(a) $v(t) =$ ______________________________

(b) $s(t) =$ ______________________________

(7) (10 points) Let $g(x) = \dfrac{3x^2+1}{x^2-x-6}$

(a) (4 points) Find the horizontal asymptotes, if any. Be sure to justify your work!

(b) (6 points) Find all vertical asymptotes. Find the left- and right-hand limits of g at each vertical asymptote. Be sure to justify your work!

(a) Horizontal asymptote(s) ____________________

(b) Vertical asymptote(s) ____________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (15 points)

(a) (5 points) State the Mean Value Theorem. Be sure to state all hypotheses as well as the conclusion.

(b) (5 points) Professor Perry drove from Lexington to Columbus, Ohio (a distance of 200 miles) on interstate highways where the speed limit is 65 miles per hour. On arrival in Columbus, he proudly told a former Calculus I student who is now a police officer that he had made the drive in only 2.5 hours. The officer promptly gave him a speeding ticket. Using the Mean Value Theorem, explain how the police officer knew that Perry had been speeding.

(c) (5 points) Suppose that f is a function continuous on $[0, 3]$, differentiable on $(0, 3)$, that $-1 \leq f'(x) \leq 4$, and $f(0) = 5$. Based on this information, is it possible that $f(3) = 20$? Explain why or why not using the Mean Value Theorem.

(9) (15 points)

(a) (5 points) State L'Hospital's Rule.

(b) (5 points) Find a and b so that

$$\lim_{x \to 0} \frac{e^{4x} - a - bx}{x^2}$$

exists (that is, is finite), and find the value of the limit for these numbers a and b.

$a =$ ____________, $b =$ ____________, limit = ____________

(c) (5 points) Use L'Hospital's rule to determine

$$\lim_{x\to 0} \frac{\sin(3x)}{\tan(6x)}.$$

The limit is ______________________

(10) (15 points) A box with a square base and an open top is to have a volume of 180cm^3. Material for the base costs \$5 per cm^2 and material for the sides costs \$3 per cm^2. Find the dimensions of the cheapest box.

Dimensions are ____________________________________