

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Last 4 digits of student ID #: \_\_\_\_\_

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

**On the free response problems:**

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

**Multiple Choice Answers**

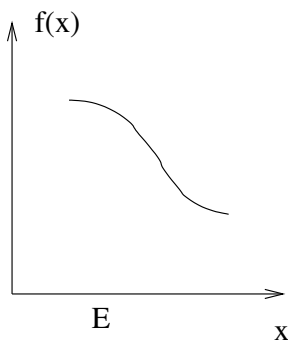
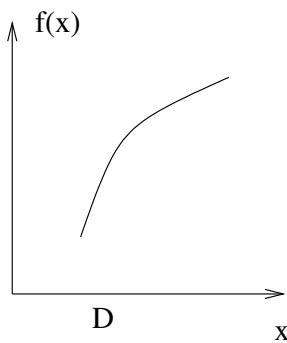
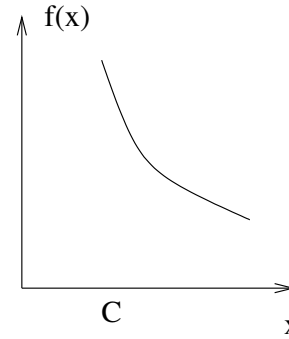
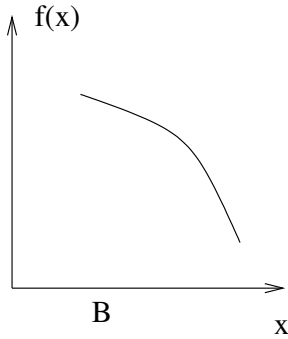
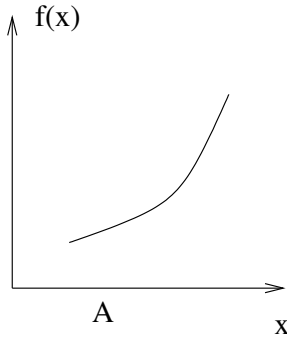
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

**Exam Scores**

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of the exam.

1. The function  $f$  satisfies  $f'(x) < 0$  for all  $x$  and  $f''(x) < 0$  for all  $x$ . Which of the following could be the graph of  $f$ ?



2. Let the function  $f$  be defined by  $f(x) = |\sin(x)|$ . How many critical points does the function  $f$  have on the open interval  $(\pi/4, 5\pi/4)$ ?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

**Record the correct answer to the following problems on the front page of the exam.**

3. Let  $f$  be a function whose domain is the interval  $(0, 2)$  and assume that  $f$  is differentiable on  $(0, 2)$ . Select the statement that must be true for any such  $f$ .
- A. If  $f$  has a local maximum at 1, then  $f'(1) = 0$ .
  - B. If  $f'(1) = 0$ , then  $f$  has a local maximum at 1.
  - C. If  $f(1) = 2012$ , then  $f$  has a local maximum at 1.
  - D. If  $f''(1) > 0$ , then  $f$  has a local minimum at 1.
  - E. If  $f(1/2) = f(3/2)$ , then  $f'(1) = 0$ .
4. Suppose that  $f$  is a function whose domain is the open interval  $(-1, 1)$  and  $f$  has a local maximum at 0. One of the statements below can never be true for  $f$ . The other four will be true for some choices of  $f$  and false for other choices. Which of the following can never be true for any such  $f$ ?
- A.  $f'(0) = 0$  and  $f''(0) < 0$
  - B.  $f'(0) = 0$  and  $f''(0) = 0$
  - C.  $f'(0)$  does not exist
  - D.  $f'(0) = 0$  and  $f'(x) < 0$  for  $0 < x < 1$
  - E.  $f'(0) = 0$  and  $f'(x) > 0$  for  $0 < x < 1$

**Record the correct answer to the following problems on the front page of the exam.**

5. Suppose that  $f$  is a function on the open interval  $(0, 3)$  and we know the following information about the derivative  $f'$ :

$$\begin{aligned} f'(x) &> 0, & 0 < x < 1 \\ f'(1) &= 0, \\ f'(x) &< 0, & 1 < x < 2 \\ f'(2) &= 0, \\ f'(x) &< 0, & 2 < x < 3 \end{aligned}$$

Which of the following is true?

- A. The function  $f$  has a local maximum at 1 and a local minimum at 2.
  - B. The function  $f$  has a local maximum at 1 and a local maximum at 2.
  - C. The function  $f$  has a local minimum at 1 and a local minimum at 2.
  - D. The function  $f$  has a local minimum at 1 and a local maximum at 2.
  - E. The function  $f$  has a local maximum at 1 and does not have an extremum at 2.
6. We let  $f(x) = x^2 - 8$  and use Newton's method to find a solution of  $f(x) = 0$ . If  $x_1 = 4$ , find the exact value of  $x_3$ .
- A.  $\sqrt{8}$
  - B. 8
  - C. 3
  - D.  $17/6$
  - E. None of the above.

**Record the correct answer to the following problems on the front page of the exam.**

7. Let  $(a, f(a))$  be the point where the tangent line to the graph of  $f(x) = x \cos(x)$  is horizontal. We use Newton's method to find  $x_1, x_2, x_3, \dots$ , the successive approximations to  $a$ . Give the formula that we use to compute  $x_{n+1}$  from  $x_n$ .

A.  $x_{n+1} = x_n - \frac{x_n \cos(x_n)}{\cos(x_n) - x_n \sin(x_n)}$

B.  $x_{n+1} = x_n - \frac{\cos(x_n) - x_n \sin(x_n)}{x_n \cos(x_n)}$

C.  $x_{n+1} = x_n + \frac{\cos(x_n)}{\sin(x_n)}$

D.  $x_{n+1} = x_n + \frac{2 \sin(x_n) + x_n \cos(x_n)}{\cos(x_n) - x_n \sin(x_n)}$

E.  $x_{n+1} = x_n + \frac{\cos(x_n) - x_n \sin(x_n)}{2 \sin(x_n) + x_n \cos(x_n)}$

8. Which of the following is **not** an anti-derivative of  $2 \sin(x) \cos(x)$ ?

A.  $1 + \sin^2(x)$

B.  $\sin^2(x)$

C.  $1 - \cos^2(x)$

D.  $-\cos^2(x)$

E.  $\sin^2(x) + \cos^2(x)$

**Record the correct answer to the following problems on the front page of the exam.**

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9. We have a sequence of numbers  $\{a_1, a_2, a_3, \dots\}$  so that  $\sum_{k=1}^n a_k = n^2$  holds for  $n = 1, 2, 3, \dots$ .

Find the value of  $\sum_{k=1}^{12} (4a_k + 2)$ .

- A. 578
- B. 600
- C. 2306
- D. 2328
- E. 2400
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10. Let  $f(x) = 3 - x$ . Subdivide the interval  $[1, 3]$  into four equal sub-intervals and compute  $R_4$ , the value of the right-endpoint approximation to the area under the graph of  $f$  on the interval  $[1, 3]$ .
- A.  $5/2$
- B. 2
- C.  $3/2$
- D. 1
- E. None of the above.

**Free Response Questions: Show your work!**

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11. The volume of a sphere of radius  $r$  is  $V(r) = 4\pi r^3/3$ .
- (a) Find  $L(r)$ , the linearization of the volume  $V$  at  $r = 10$ .
  - (b) A sphere has radius of 10 centimeters. The sphere is heated and the radius increases 6%. Use the linearization to approximate the increase in the volume of the sphere. Please give your answer as a multiple of  $\pi$ .

**Free Response Questions: Show your work!**

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**12.** Let  $f(x) = x^3 + 3x^2 - 9x + 2012$ .

(a) Find the open intervals where  $f$  is increasing or decreasing.

(b) Find the intervals where  $f$  is concave up or concave down.

(c) Give all inflection points of  $f$ .

Use calculus to justify your answers.



**Free Response Questions: Show your work!**

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13. Evaluate the following limits. Explain your reasoning.

(a)  $\lim_{x \rightarrow 1} \frac{\ln(x)}{x^2 - 1}$ .

(b)  $\lim_{x \rightarrow 1} \frac{e^x}{x^2 - 2}$

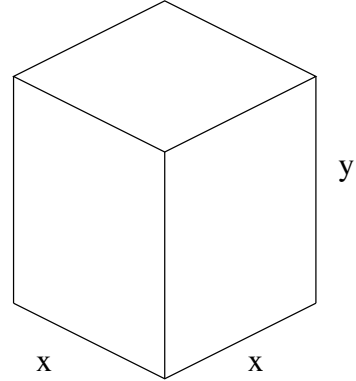
(c)  $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{x^2}$

**Free Response Questions: Show your work!**

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14. A large shipping box has a square base of sidelength  $x$  meters. The height of the container is  $y$  meters. Central Associated Transport Services (CATS) will only accept the box if the sum of the height and the perimeter of the base is equal to 10 meters.

- (a) Write down a function which gives the volume of the box in terms of  $x$ , the sidelength of the base. Give the domain of the function which gives the volume.
- (b) Find the dimensions  $x$  and  $y$  of the box with the **largest** possible volume.
- (c) Explain how we know we have found the largest possible volume.



**Free Response Questions: Show your work!**

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15. Let  $g$  be a twice differentiable function and suppose that  $g''(x) = 3x + \cos(x)$ . If  $g'(0) = 2$  and  $g(1) = 3$ , find the function  $g$ .