

Multiple Choice Answers

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Give the linearization of the function $f(x) = x \sin x$ at the point $x = \pi$:
 - (A) $y(x) = \pi - \pi x$
 - (B) $y(x) = \pi^2 - \pi x$
 - (C) $y(x) = \pi^2 + \pi x$
 - (D) $y(x) = x - \pi$
 - (E) $y(x) = \pi x$

2. Let f be a differentiable function. Newton's method provides a sequence of successive approximations to a root x^* of the equation $f(x) = 0$. Suppose x_0 is the first guess, and that x_1 and x_2 are computed according to Newton's method. What is the formula for the third approximation x_3 to the root x^* ?
 - (A) $x_3 = x_0 + f(x_2)/f'(x_2)$
 - (B) $x_3 = x_1 - f(x_2)/f'(x_2)$
 - (C) $x_3 = x_2 - f(x_2)/f'(x_2)$
 - (D) $x_3 = x_2 - f(x_1)/f'(x_1)$
 - (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

3. The indefinite integral $\int x^{-1/3} dx$ is equal to

- (A) $3x^{1/3} + C$
- (B) $x^{2/3} + C$
- (C) $\frac{2}{3}x^{1/3} + C$
- (D) $\frac{3}{2}x^{2/3} + C$
- (E) None of the above

4. If $\sum_{j=1}^{10} a_j = 20$ and $\sum_{j=1}^{10} b_j = 44$, then the sum

$$\sum_{j=1}^{10} (5a_j - 2b_j)$$

equals

- (A) 12
- (B) 60
- (C) 88
- (D) 100
- (E) 188

Record the correct answer to the following problems on the front page of this exam.

5. The most general anti-derivative of $f(x) = x^2 + \cos(\pi x)$ is:

- (A) $x^3 + \sin(\pi x) + C$
- (B) $x - \pi \sin(\pi x) + C$
- (C) $\frac{1}{3}x^3 + \frac{1}{\pi} \sin(\pi x) + C$
- (D) $\frac{1}{3}x^3 - \sin(\pi x)$
- (E) $\frac{1}{3}x^3 + \pi \cos(\pi x)$

6. The limit

$$\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 7x + 2}{-2x^2 - 4x + 3} \right)$$

is:

- (A) $3/2$
- (B) $2/3$
- (C) $-3/2$
- (D) $7/4$
- (E) Does not exist

Record the correct answer to the following problems on the front page of this exam.

7. The function $f(x) = x^3 - 6x^2 + 8x$ is:

- (A) concave up on $(-\infty, 0)$ and on $(2, \infty)$
- (B) concave up on $(0, 2)$
- (C) concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$
- (D) concave down on $(2, 4)$
- (E) concave up on $(-\infty, 2)$ and concave down on $(2, \infty)$

8. The limit

$$\lim_{x \rightarrow 1} \left(\frac{x^2 - 1}{\sin(\pi x)} \right)$$

is:

- (A) -2
- (B) $2/\pi$
- (C) -2π
- (D) $-2/\pi$
- (E) Does not exist

Record the correct answer to the following problems on the front page of this exam.

9. The critical points of the function $f(x) = x^2(x^2 - 4)$ are

- (A) $\{-2, 0, 2\}$
- (B) $\{-\sqrt{2}, \sqrt{2}\}$
- (C) $\{-\sqrt{2}, 0, \sqrt{2}\}$
- (D) $\{-1, 0, 1\}$
- (E) $\{-\sqrt{2}/2, 0, \sqrt{2}/2\}$

10. A baseball is thrown upward and its height in meters from the ground satisfies $h(t) = 20t - 5t^2$, where time is measured in seconds. At what time is the ball's instantaneous velocity equal to its average velocity over the time interval $[0, 2]$?

- (A) 10 seconds
- (B) 2 seconds
- (C) 1 second
- (D) 1/2 second
- (E) 4 seconds

Free Response Questions: Show your work!

11. An adventurer wants to reach a camp 10 km downstream on the opposite side of a perfectly straight 2 km wide river. Starting at the edge of the river, the adventurer swims at 2 km/hr in a straight line to a point on the opposite shore. The adventurer then walks at 4 km/hr the remainder of the way to the camp.
- (a) Sketch the situation labeling all important quantities.
- (b) Find the path that minimizes the time it takes the adventurer to get to the camp 10 km downstream on the opposite side. How many kilometers downstream from his starting point will the adventurer be when he finishes his swim and reaches the opposite shore? Present all your work.

Free Response Questions: Show your work!

12. Find the critical points of the function f on the interval $(-2, 2)$ where

$$f(x) = \begin{cases} 2x & -2 < x < 0 \\ x^2 - 2x & 0 \leq x < 2 \end{cases}$$

Determine if the function f has a local maximum or a local minimum at each critical point. Show all your work and use calculus to justify your answers.

Free Response Questions: Show your work!

13. (a) State the Mean Value Theorem.
- (b) Suppose that f is a differentiable function on the real line and $2 \leq f'(x) \leq 4$ for x in the interval $(1, 6)$. If $f(1) = 2$, use the Mean Value Theorem for f on the interval $[1, 6]$ to determine the largest and smallest possible values for $f(6)$.

Free Response Questions: Show your work!

14. (a) We know the following sum for $N = 1, 2, 3, \dots$:

$$\sum_{j=1}^N j = \frac{N(N+1)}{2}.$$

Using this, evaluate the sum

$$\frac{1}{N} \sum_{j=1}^N \left(5 + \frac{2j}{N} \right)$$

- (b) Compute the limit as N tends to infinity of the result obtained in part a).

Free Response Questions: Show your work!

15. The graph below is the graph of the function $f(x) = x^3 - 5x + 1$.

- (a) Taking $x_0 = 2.5$, use Newton's method to find a root of the equation $f(x) = 0$. Show your work below and give the values of x_1 and x_2 correctly rounded to three decimal places in the table to the right.

n	x_n
0	2.500
1	2.200
2	2.132

- (b) Taking $x_0 = -3$, use Newton's method to find a root of the equation $f(x) = 0$. Show your work below and give the values of x_1 and x_2 correctly rounded to three decimal places in the table to the right.

n	x_n
0	-3.000
1	-2.500
2	-2.345