MA 113 Calculus I — Fall 2014

Exam 3 — Tuesday, 18 Novemeber 2014

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

B, C, E, C, B, B, D, B, E, E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Let $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x$. Which of the following statements must be true?

   (A) f has critical points at -2, 5 and $\frac{3}{2}$.

   (B) f has critical points at -2 and 5, and an inflection point at $\frac{3}{2}$.

   (C) f has critical points at -2 and 5, but has no infection points.

   (D) f has an inflection point at $\frac{3}{2}$, but has no critical points.

   (E) f has no critical points and no inflection points.

2. The linear approximation to $f(x) = \sqrt{x^2 + 3}$ at $x = 1$ is

   (A) $L(x) = x + 2$

   (B) $L(x) = \frac{1}{2}x + \frac{1}{2}$

   (C) $L(x) = \frac{1}{2}x + \frac{3}{2}$

   (D) $L(x) = \frac{1}{4}x + \frac{1}{2}$

   (E) $L(x) = x + \frac{1}{2}$

3. $f(x)$ is differentiable on $(0, 2)$ and $f(0) = 1$ and $f(2) = 5$. Which of the following statements must be true?

   (A) $f(x)$ is increasing on $(0, 2)$.

   (B) There is a point c in $(0, 2)$ such that $x = c$ is a critical point.

   (C) There is a point c in $(0, 2)$ such that $x = c$ is a local max or a local min.

   (D) There is a point c in $(0, 2)$ such that $x = c$ is an inflection point.

   (E) There is a point c in $(0, 2)$ such that $f'(c) = 2$.

Record the correct answer to the following problems on the front page of this exam.

4. Find $\lim_{x \to +\infty} \dfrac{(4x-2)(x+3)(x-4)}{(2x+3)(x+1)(x-5)}$.

   (A) 0

   (B) 1

   (C) 2

   (D) 4

   (E) $+\infty$

5. The value of $\lim_{x \to 0^+} x^{\frac{1}{2}} \ln x$ is

   (A) $-\infty$

   (B) 0

   (C) $\frac{1}{2}$

   (D) 1

   (E) $-\infty$

Record the correct answer to the following problems on the front page of this exam.

6. Find $\lim_{t \to 0} \frac{(e^{t^2} - 1)}{t^2}$.

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) $+\infty$

7. Let $f(x) = x^2 e^x - 1$. We want to use Newton's method to solve $f(x) = 0$. Let $x_0 = 1$. Find x_1 rounded to the nearest 3 digits after the decimal point.

   (A) 0.143

   (B) 0.332

   (C) 0.511

   (D) 0.789

   (E) 0.923

Record the correct answer to the following problems on the front page of this exam.

8. Assume that the functions $f(x)$ and $f'(x)$ are each differentiable everywhere. If a and b are two critical points of $f(x)$ (where $a < b$) and $f(x)$ is not constant on (a, b), which of the following statements must be true?

   (A) $f(x)$ has at least one additional critical point in (a, b).

   (B) $f(x)$ has at least one inflection point in (a, b).

   (C) The absolute maximum of $f(x)$ on $[a, b]$ occurs at either a or b.

   (D) $f(x)$ has either a local max or a local min in (a, b).

   (E) $f(x)$ has no additional critical points in (a, b).

9. Given that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, find $\sum_{k=10}^{15}(2k + 3)$.

   (A) 148

   (B) 152

   (C) 158

   (D) 162

   (E) 168

10. The antiderivative $\int x^2 + \sin(2x - 7)\, dx$ is equal to

    (A) $x^3 - \cos(2x - 7) + C$

    (B) $\frac{1}{3}x^3 - 2\cos(2x - 7) + C$

    (C) $\frac{1}{3}x^3 + 2\cos(2x - 7) + C$

    (D) $\frac{1}{3}x^3 + \frac{1}{2}\cos(2x - 7) + C$

    (E) $\frac{1}{3}x^3 - \frac{1}{2}\cos(2x - 7) + C$

Free Response Questions: Show your work!

11. In parts (a) and (b), let $f(x) = \tan x$.

    (a) Find $L(x)$ where $L(x)$ is the linear approximation of $f(x) = \tan x$ at $x = 0$.

    $L(x) = f(0) + f'(0)(x - 0) = x$ because $\tan(0) = 0$, $\tan(x)' = \sec^2(x)$, and $\sec(0) = 1$.

    (b) Compute $\lim_{x\to 0} \dfrac{f(x) - L(x)}{x}$.

    $$\lim_{x\to 0} \frac{f(x) - L(x)}{x} = \lim_{x\to 0} \frac{\tan(x) - x}{x} = \lim_{x\to 0} \frac{\sec^2(x) - 1}{1} = \frac{1 - 1}{1} = 0.$$

Free Response Questions: Show your work!

12. A manufacturer wants to design an open box that has a square base, no top, and a surface area of 108 square inches.

    (a) Sketch the box and label its dimensions.

    Let x denote the length of a side of the square base. Let h denote the height of the box. Let S denote the surface area of the box and let V denote the volume of the box.

    (b) Write the equation stating that the surface area of the box is 108 square inches.

    $S = x^2 + 4xh = 108$.

    (c) What dimensions will produce a box with the maximum volume?

    $V = x^2 h = x^2(\frac{108-x^2}{4x}) = \frac{108x-x^3}{4}$. We set $V'(x) = \frac{108-3x^2}{4} = 0$. This gives $x = 6$ (because $x > 0$). The domain of x is $(0, \sqrt{108}]$. The volume is zero when $x = \sqrt{108}$. $V''(6) < 0$. Thus the maximum must occur when $x = 6$. In that case $h = 3$.

Free Response Questions: Show your work!

13. Let $f(x) = -x^4 + 4x^3 + 4353$.

    (a) Find the intervals over which $f(x)$ is increasing, and find the intervals over which $f(x)$ is decreasing. Find all local minima and all local maxima.

    (b) Find the intervals over which $f(x)$ is concave up, and find the intervals over which $f(x)$ is concave down. Find all inflection points.

Free Response Questions: Show your work!

14. The acceleration of a particle moving along the x-axis is given by the equation $a(t) = -3t^2 + 2t$. At $t = 0$, the particle is at $x = 1$ with a velocity of 5 m/s.

    (a) Find the velocity function $v(t)$ of this particle and use it to find $v(2)$.

    (b) Find the position function $s(t)$ of this particle and use it to find $s(2)$.

Free Response Questions: Show your work!

15. Consider the function $f(x) = x^2$. We are interested in the area A that is under the graph of $f(x)$ and above the interval $[0, 4]$.

    (a) Divide the interval $[0, 4]$ into n intervals of equal length and write the expression for R_n, the sum that represents the right-endpoint approximation to the area A.

    (b) Use the fact that $\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$ to find a closed form expression for R_n.

    (c) Take the limit of R_n as n tends to infinity to find an exact value for the area A.