MA 113 Calculus I   Fall 2014
Exam 3   Tuesday, 18 Novemeber 2014

Name: ———————————————

Section: ———————————————

Last 4 digits of student ID #: ————

This exam has ten multiple choice questions
(five points each) and five free response ques-
tions (ten points each). Additional blank sheets
are available if necessary for scratch work. No
books or notes may be used. Turn off your
cell phones and do not wear ear-plugs during
the exam. You may use a calculator, but not
one which has symbolic manipulation capa-
bilities.

On the multiple choice problems:

1. You must give your *final answers* in the
   multiple choice answer box on the front
   page of your exam.

2. Carefully check your answers. No credit
   will be given for answers other than those
   indicated on the *multiple choice answer
   box*.

On the free response problems:

1. Clearly indicate your answer and the rea-
   soning used to arrive at that answer (*un-
   supported answers may not receive credit*),

2. Give exact answers, rather than decimal
   approximations to the answer (unless oth-
   erwise stated).

Each free response question is followed by
space to write your answer. Please write your
solutions neatly in the space below the ques-
tion. You are not expected to write your so-
lution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

1. Let $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x$. Which of the following statements must be true?

   (A) f has critical points at -2, 5 and $\frac{3}{2}$.

   (B) f has critical points at -2 and 5, and an inflection point at $\frac{3}{2}$.

   (C) f has critical points at -2 and 5, but has no inflection points.

   (D) f has an inflection point at $\frac{3}{2}$, but has no critical points.

   (E) f has no critical points and no inflection points.

2. The linear approximation to $f(x) = \sqrt{x^2 + 3}$ at $x = 1$ is

   (A) $L(x) = x + 2$

   (B) $L(x) = \frac{1}{2}x + \frac{1}{2}$

   (C) $L(x) = \frac{1}{2}x + \frac{3}{2}$

   (D) $L(x) = \frac{1}{4}x + \frac{1}{2}$

   (E) $L(x) = x + \frac{1}{2}$

3. $f(x)$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$. Assume that $f(0) = 1$ and $f(2) = 5$. Which of the following statements must be true?

   (A) $f(x)$ is increasing on $(0, 2)$.

   (B) There is a point c in $(0, 2)$ such that $x = c$ is a critical point.

   (C) There is a point c in $(0, 2)$ such that $x = c$ is a local max or a local min.

   (D) There is a point c in $(0, 2)$ such that $x = c$ is an inflection point.

   (E) There is a point c in $(0, 2)$ such that $f'(c) = 2$.

4. Find $\lim\limits_{x \to +\infty} \dfrac{(4x-2)(x+3)(x-4)}{(2x+3)(x+1)(x-5)}$.

    (A) 0

    (B) 1

    (C) 2

    (D) 4

    (E) $+\infty$

5. The value of $\lim\limits_{x \to 0^+} x^{\frac{1}{2}} \ln x$ is

    (A) $-\infty$

    (B) 0

    (C) $\frac{1}{2}$

    (D) 1

    (E) ∞

6. Find $\lim\limits_{t \to 0} \dfrac{(e^{t^2} - 1)}{t^2}$.

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) $+\infty$

7. Let $f(x) = x^2 e^x - 1$. We want to use Newton's method to solve $f(x) = 0$. Let $x_0 = 1$. Find x_1 rounded to the nearest 3 digits after the decimal point.

   (A) 0.143

   (B) 0.332

   (C) 0.511

   (D) 0.789

   (E) 0.923

8. Assume that the functions $f(x)$ and $f'(x)$ are each differentiable everywhere. If a and b are two critical points of $f(x)$ (where $a < b$) and $f(x)$ is not constant on (a, b), which of the following statements must be true?

   (A) $f(x)$ has at least one additional critical point in (a, b).

   (B) $f(x)$ has at least one inflection point in (a, b).

   (C) The absolute maximum of $f(x)$ on $[a, b]$ occurs at either a or b.

   (D) $f(x)$ has either a local max or a local min in (a, b).

   (E) $f(x)$ has no additional critical points in (a, b).

9. Given that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, find $\sum_{k=10}^{15}(2k + 3)$.

   (A) 148

   (B) 152

   (C) 158

   (D) 162

   (E) 168

10. The antiderivative $\int x^2 + \sin(2x - 7)\, dx$ is equal to

   (A) $x^3 - \cos(2x - 7) + C$

   (B) $\frac{1}{3}x^3 - 2\cos(2x - 7) + C$

   (C) $\frac{1}{3}x^3 + 2\cos(2x - 7) + C$

   (D) $\frac{1}{3}x^3 + \frac{1}{2}\cos(2x - 7) + C$

   (E) $\frac{1}{3}x^3 - \frac{1}{2}\cos(2x - 7) + C$

11. In parts (a) and (b), let $f(x) = \tan x$.

    (a) Find $L(x)$ where $L(x)$ is the linear approximation of $f(x) = \tan x$ at $x = 0$.

    (b) Compute $\displaystyle\lim_{x \to 0} \frac{f(x) - L(x)}{x}$.

12. A manufacturer wants to design an open box that has a square base, no top, and a surface area of 108 square inches.

    (a) Sketch the box and label its dimensions.

    (b) Write the equation stating that the surface area of the box is 108 square inches.

    (c) What dimensions will produce a box with the maximum volume?

13. Let $f(x) = -x^4 + 4x^3 + 4353$.

    (a) Find the intervals over which $f(x)$ is increasing, and find the intervals over which $f(x)$ is decreasing. Find all local minima and all local maxima.

    (b) Find the intervals over which $f(x)$ is concave up, and find the intervals over which $f(x)$ is concave down. Find all inflection points.

14. The acceleration of a particle moving along the x-axis is given by the equation $a(t) = -3t^2 + 2t$. At $t = 0$, the particle is at $x = 1$ with a velocity of 5 m/s.

    (a) Find the velocity function $v(t)$ of this particle and use it to find $v(2)$.

    (b) Find the position function $s(t)$ of this particle and use it to find $s(2)$.

15. Consider the function $f(x) = x^2$. We are interested in the area A that is under the graph of $f(x)$ and above the interval $[0, 4]$.

    (a) Divide the interval $[0, 4]$ into n intervals of equal length and write the expression for R_n, the sum that represents the right-endpoint approximation to the area A.

    (b) Use the fact that $\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$ to find a closed form expression for R_n.

    (c) Take the limit of R_n as n tends to infinity to find an exact value for the area A.