MA 113 Calculus I — Fall 2015
Exam 3 — Tuesday, 17 November 2015

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-phones during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

C,D,B,C,D, C,A,D,B,A

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

1. Find an antiderivative of $4e^{2x+1}$.

   (A) $8e^{2x+1}$

   (B) $4(2x+1)e^{2x+1}$

   (C) $2e^{2x+1}$

   (D) $4e^{2x+1}+1$

   (E) $4xe^{2x+1}+1$

2. A function $f(x)$ is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b). Which of the following statements is always true?

   (A) The absolute maximum value of f occurs at a point x where $f'(x) = 0$.

   (B) $f(x)$ has at least one point of inflection on the interval $[a, b]$.

   (C) $f(x)$ has at least one local minimum on the open interval (a, b).

   (D) $f(x)$ has at least one absolute maximum value on the interval $[a, b]$.

   (E) $f(x)$ has at least one critical point on the interval (a, b).

Record the correct answer to the following problems on the front page of this exam.

3. Evaluate the indefinite integral $\int (3\sec^2 x - \frac{2}{\sqrt{x}} + 7x^{4/3})\,dx$.

   (A) $\sec^3 x + 4x^{1/2} + 3x^{7/3} + C$

   (B) $3\tan x - 4x^{1/2} + 3x^{7/3} + C$

   (C) $3\tan x + 4x^{1/2} + 3x^{7/3} + C$

   (D) $\sec^3 x - 4x^{1/2} + \frac{3}{7}x^{7/3} + C$

   (E) None of the above.

4. A particle moves along the x-axis with velocity $v(t) = 3t^2 + 4t + 2$ meters/second. Find the particle's position $s(3)$ assuming that $s(2) = 25$.

   (A) 54

   (B) 55

   (C) 56

   (D) 57

   (E) 58

Record the correct answer to the following problems on the front page of this exam.

5. Evaluate the limit $\lim_{x \to \infty} \dfrac{6x^2 - 3x + 4}{\sqrt{4x^4 + 2x^2 + 1}}$.

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) 4

6. We approximate $\sqrt[4]{113}$ by solving $x^4 - 113 = 0$ with Newton's method. Which of the following relations is iterated? (Suggestion: Write down Newton's method and simplify the algebraic expression.)

   (A) $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{113}{x_n^3}\right)$

   (B) $x_{n+1} = \dfrac{x_n^4 - 113}{4x_n^3}$

   (C) $x_{n+1} = \dfrac{3}{4}x_n + \dfrac{113}{4x_n^3}$

   (D) $x_{n+1} = \dfrac{1}{4}x_n - \dfrac{113}{x_n^3}$

   (E) $x_{n+1} = x_n + \dfrac{113}{x_n^3}$

Record the correct answer to the following problems on the front page of this exam.

7. Evaluate the limit $\lim_{x \to 0} \dfrac{\ln(1-2x)}{x}$.

   (A) -2

   (B) -1

   (C) 0

   (D) 1

   (E) 2

8. You are given a function $f(x)$ such that the derivative $f'(x) = \dfrac{(x-1)^2(x+1)}{x^2+1}$. Which of the following statements is correct?

   (A) $f(-1)$ and $f(1)$ are both local minimums.

   (B) $f(-1)$ is a local minimum and $f(1)$ is a local maximum.

   (C) $f(-1)$ and $f(1)$ are both local maximums.

   (D) $f(-1)$ is a local minimum and $f(1)$ is neither a local maximum nor a local minimum.

   (E) $f(-1)$ is neither a local maximum nor a local minimum, and $f(1)$ is a local minimum.

Record the correct answer to the following problems on the front page of this exam.

9. Consider the function $f(x) = x + 2 + \dfrac{3}{x-1}$. Which one of the following statements is correct?

   (A) The function $f(x)$ is concave up everywhere.
   (B) The function $f(x)$ is concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$.
   (C) The function $f(x)$ is concave down on $(1, \infty)$ and concave up on $(-\infty, 1)$.
   (D) The function $f(x)$ is concave down on $(1, \infty)$ and concave down on $(-\infty, 1)$.
   (E) The function $f(x)$ is concave down everywhere.

10. Suppose that the right-endpoint approximation for computing the area under the graph of some function results in the following expression.

$$R_N = \frac{4}{N} \sum_{j=1}^{N} \left(\frac{6j^3}{N^3} + \frac{8j^2}{N^2} \right).$$

   Find $\lim_{N \to \infty} R_N$. (You may use the formulas given in Problem 13.)

   (A) $\dfrac{50}{3}$
   (B) $\dfrac{52}{3}$
   (C) $\dfrac{54}{3}$
   (D) $\dfrac{56}{3}$
   (E) $\dfrac{58}{3}$

11. What is the maximum possible area of a rectangle with base that lies on the x-axis and with the two upper vertices that lie on the graph $y = 9 - x^2$ (Be sure to justify your answer.)

    Let (x, y) denote the vertex of the rectangle on the parabola in the first quadrant and let $A(x)$ denote the area of the rectangle. Then $A(x) = 2x(9 - x^2) = 18x - 2x^3$. The domain of $A(x)$ is $0 \le x \le 3$. $A'(x) = 18 - 6x^2 = 0$. $x^2 = 3$, $x = \sqrt{3}$. $A''(x) = -12x$, so $A''(\sqrt{3}) < 0$. Thus the absolute maximum of $A(x)$ is $A(\sqrt{3}) = 2\sqrt{3}(9 - 3) = 12\sqrt{3}$. $(A(0) = A(3) = 0.)$

Free Response Questions: Show your work!

12. (a) Find the linearization of the function $f(x) = \sqrt{x}$ at $x = 4$.

$f'(x) = \dfrac{1}{2\sqrt{x}}$. The linearization of $f(x)$ at $x = 4$ is

$$L(x) = f(4) + f'(4)(x - 4) = 2 + \frac{1}{4}(x - 4) = \frac{1}{4}x + 1.$$

(b) Use part (a) to approximate $\sqrt{4.0804}$. (You will not receive credit for giving the exact value of $\sqrt{4.0804}$.)

$\sqrt{4.0804} = f(4.0804) \approx L(4.0804) = 1 + 1.0201 = 2.0201$. (The exact value of $\sqrt{4.0804}$ is 2.02.)

13. Consider the function $f(x) = x^2$ on the interval $[0, 5]$. Subdivide the interval $[0, 5]$ into 10 intervals of equal length. Recall that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

(a) Approximate the area under the curve $f(x) = x^2$ above the x-axis on this interval by using the right-endpoint approximation.

$\Delta x = \dfrac{5-0}{10} = \dfrac{1}{2}$. Then

$$R_{10} = \Delta x \sum_{j=1}^{10} f(j\Delta x) = \frac{1}{2}\sum_{j=1}^{10}\left(\frac{j}{2}\right)^2 = \frac{1}{8}\sum_{j=1}^{10} j^2 = \frac{1}{8}\cdot\frac{10\cdot 11\cdot 21}{6} = \frac{385}{8} = 48\frac{1}{8}.$$

(b) Is this an under-estimate or an over-estimate of the area? Give geometric reasoning. (No credit will be given if you apply the Fundamental Theorem of Calculus, because we have not yet covered this result in class.)

The result in part (a) is an over-estimate because f is an increasing function on $[0, 5]$, and so the right-endpoint approximation gives rectangles that lie above the graph of the function.

Free Response Questions: Show your work!

14. Consider the function $f(x) = -x^3 + 3x^2 + 24x - 113$. Use Calculus methods to solve the following problems. Be sure to show your work to explain how you obtained your answers.

    (a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

    $f'(x) = -3x^2 + 6x + 24 = -3(x^2 - 2x - 8) = -3(x-4)(x+2)$. If $f'(x) = 0$, then $x = -2, 4$. Take a test point in each of the intervals $(-\infty, -2)$, $(-2, 4)$, and $(4, \infty)$ to conclude that $f(x)$ is increasing in the interval $(-2, 4)$ and $f(x)$ is decreasing in the intervals $(-\infty, -2)$ and $(4, \infty)$.

    (b) Find the intervals where the graph of $f(x)$ is concave up and the intervals where the graph of $f(x)$ is concave down.

    $f''(x) = -6x + 6 = -6(x - 1)$. Take a test point in each of the intervals $(-\infty, 1)$ and $(1, \infty)$ to conclude that the graph of $f(x)$ is concave up in the interval $(-\infty, 1)$ and concave down in the interval $(1, \infty)$.

Free Response Questions: Show your work!

15. (a) Carefully state the Mean Value Theorem.

Suppose that $f(x)$ is a continuous function on the closed interval $[a, b]$ and a differentiable function on the open interval (a, b). Then there exists at least one value $c \in (a, b)$ such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

(b) For the function $f(x) = x^3$ and the interval $[-1, 5]$, find the point guaranteed by the Mean Value Theorem.

$3c^2 = f'(c) = \dfrac{f(5) - f(-1)}{5 - (-1)} = \dfrac{126}{6} = 21$. Then $c^2 = 7$ and so $c = \sqrt{7}$. ($-\sqrt{7} \notin (-1, 5)$.) The point is $(\sqrt{7}, \sqrt{7}^3)$.