MA 113 Calculus I Fall 2016
Exam 3 Tuesday, November 15, 2016

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. True or False: The integral $\int_{-3\pi}^{\pi} \sin x \, dx$ equals zero.

2. True or False: There is no function which is always increasing and has exactly two points of inflection.

3. True or False: If $\int_{-2}^{10} f(x)\, dx = 25$ and $\int_{-2}^{5} f(x)\, dx = 17$ then $\int_{5}^{10} 3f(x)\, dx = 25$.

4. True or False: If the doubling time of a colony of bacteria is 4 years, then the time for this colony of bacteria to be eight times its current size is 12 years.

5. True or False: In summation notation $1+3+5+7+9+11+13+15+17$ is $\sum_{k=0}^{8}(2k+1)$.

6. The solution to

$$\frac{dy}{dt} = 5y$$

satisfying $y(6) = 9$ is

(A) $y = \frac{9}{e^{30}}e^{-5t}$

(B) $y = \frac{9}{e^{30}}e^{5t}$

(C) $y = \frac{9}{e^{6}}e^{t}$

(D) $y = 3 + \frac{6}{e^{36}}e^{t}$

(E) None of the above

Record the correct answer to the following problems on the front page of this exam.

7. The absolute minimum and absolute maximum of the function $f(x) = 2x^3 - 9x^2 + 11$ on the interval $[-2, 1]$ is

   (A) Absolute minimum is -16 and absolute maximum is 11

   (B) Absolute minimum is 4 and absolute maximum is 11

   (C) Absolute minimum is -41 and absolute maximum is 4

   (D) Absolute minimum is -16 and absolute maximum is 4

   (E) Absolute minimum is -41 and absolute maximum is 11

8. Find $\lim\limits_{x\to+\infty} \dfrac{(12x+1)(x+1)(x-3)}{(3x+2)(2x-3)(1-2x)}$.

   (A) -1

   (B) 1

   (C) 4

   (D) -4

   (E) None of the above

9. The estimate of the area under the curve $f(x) = 1/x$ from $x = 1$ to $x = 2$ using four rectangles and right-hand endpoints is

   (A) 0.6931 and is an underestimate of the actual area.

   (B) 0.7595 and is an underestimate of the actual area.

   (C) 0.7595 and is an overestimate of the actual area.

   (D) 0.6345 and is an underestimate of the actual area.

   (E) 0.6345 and is an overestimate of the actual area.

Record the correct answer to the following problems on the front page of this exam.

10. Let L_n be the left-endpoint approximation for the area under the curve $f(x) = x^2$ from $x = 0$ to $x = 1$ using n rectangles. Using the fact $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, the closed form expression for L_n and its limit as n tends to infinity are

(A) $L_n = \dfrac{(n+1)(2n+1)}{6n^2}$ and $\lim\limits_{n\to+\infty} L_n = 1/3.$

(B) $L_n = \dfrac{(n+1)(2n+1)}{6n^2}$ and $\lim\limits_{n\to+\infty} L_n = 1/6.$

(C) $L_n = \dfrac{(n-1)(2n-1)}{6n^2}$ and $\lim\limits_{n\to+\infty} L_n = 1/3.$

(D) $L_n = \dfrac{(n-1)(2n-1)}{6n^2}$ and $\lim\limits_{n\to+\infty} L_n = 1/6.$

(E) None of the above

11. The expression $\lim\limits_{n\to+\infty} \sum\limits_{i=1}^{n} \sqrt{4 + (1 + 2i/n)^2} \cdot \dfrac{2}{n}$ equals

(A) $\displaystyle\int_1^3 \sqrt{4 + x^2}\, dx$

(B) $\displaystyle\int_0^2 \sqrt{4 + x^2}\, dx$

(C) $\displaystyle\int_1^3 \sqrt{4 + (1 + x)^2}\, dx$

(D) $\displaystyle\int_0^3 \sqrt{4 + (1 + x)^2}\, dx$

(E) $\displaystyle\int_0^2 \sqrt{4 + (1 + 2x)^2}\, dx$

12. Evaluate $\displaystyle\int (x + 2x^{-1} + 3x^{-2} + 4\sin x)\, dx$

(A) $\dfrac{x^2}{2} + 2\ln|x| - 3x^{-1} - 4\cos x + C$

(B) $\dfrac{x^2}{2} + 2\ln x - 3x^{-1} + 4\cos x + C$

(C) $\dfrac{x^2}{2} + 2\ln|x| - x^{-3} - 4\cos x + C$

(D) $\dfrac{x^2}{2} - x^{-2} - x^{-3} - 4\cos x + C$

(E) $\dfrac{x^2}{2} - x^{-2} - x^{-3} + 4\cos x + C$

Record the correct answer to the following problems on the front page of this exam.

13. Given that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, find $\sum_{k=16}^{20}(4k-1)$.

    (A) 365
    (B) 359
    (C) 356
    (D) 355
    (E) 354

14. Nobelium-249 has a half-life of 58 minutes. A rock sample has 100 mg of Nobelium-259 in it. How long will it take for the Nobelium-259 in the rock sample to decay to 10 mg?

    (A) 3.01 hours
    (B) 3.21 hours
    (C) 3.41 hours
    (D) 3.61 hours
    (E) 3.81 hours

15. The critical points of the function $f(\theta) = \sin\theta \cdot \cos\theta$ on the interval $[0, \pi]$ are:

    (A) 0, $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$, 2π
    (B) 0, $\pi/4$, $3\pi/4$, π
    (C) 0, $\pi/4$, π
    (D) 0, $\pi/4$, $3\pi/4$
    (E) $\pi/4$, $3\pi/4$

Free Response Questions: Show your work!

16. (a) State the Mean Value Theorem.

(b) Suppose that f is a differentiable function on the real line and $3 \leq f'(x) \leq 4$ for x in the interval $(2, 7)$. If $f(7) = 9$, use the Mean Value Theorem for f in the interval $[2, 7]$ to determine the largest and smallest possible values for $f(2)$.

Free Response Questions: Show your work!

17. Evaluate the following limits. Be sure to explain your reasoning.

(a) $\lim_{x \to +\infty} x^2 \cdot \sin \frac{\pi}{x^2}$

(b) $\lim_{x \to 0} \frac{e^{3x} - 1 - 3x}{x^2}$

Free Response Questions: Show your work!

18. Consider the function $f(x) = 3x^4 + 6x^3 - 113$. Use methods of Calculus to solve the following. Be sure to show your work and explain how you obtained your answers.

    (a) Find the interval(s) where the function $f(x)$ is increasing and the interval(s) where the function $f(x)$ is decreasing.

    (b) Find the interval(s) where the graph of $f(x)$ is concave up and the interval(s) where the graph of $f(x)$ is concave down.

Free Response Questions: Show your work!

Name: ________________________________ **Student ID Number:**__________

19. A manufacturer wishes to design an open box from a rectangular piece of cardboard having length L and width W. The original piece of cardboard has area 256 cm^2. The manufacturer forms the box by cutting out a square of sidelength 1 cm from each corner and folding up the sides to form the box.

    (a) Draw a picture of the box and label all quantities.

    (b) Write the equation stating that the area of the cardboard is 256 cm^2.

    (c) Use methods of Calculus to determine what the dimensions of the original piece of cardboard should be in order to produce a box with the maximum volume.