Exam 3

Form A

Name: _____ Section and/or TA: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**9**	A	B	C	D	E
2	A	B	C	D	E	**10**	A	B	C	D	E
3	A	B	C	D	E	**11**	A	B	C	D	E
4	A	B	C	D	E	**12**	A	B	C	D	E
5	A	B	C	D	E	**13**	A	B	C	D	E
6	A	B	C	D	E	**14**	A	B	C	D	E
7	A	B	C	D	E	**15**	A	B	C	D	E
8	A	B	C	D	E	**16**	A	B	C	D	E

SCORE

Multiple Choice	17	18	19	20	Total Score
64	9	9	9	9	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Let $f(x) = 1/x$. If possible, find the absolute maximum and minimum values for f on the interval $[2, \infty)$.

   A. The maximum is $1/2$ and the minimum does not exist.

   B. The maximum is $1/2$ and the minimum is 0.

   C. The maximum is 2 and the minimum is 0.

   D. The maximum is 2 and the minimum does not exist.

   E. The maximum does not exist and the minimum is 0.

2. The volume of a sphere is increasing at a rate of 4π cubic centimeters per minute. Find the rate of change of the radius with respect to time when the radius, r, of the sphere is 3 centimeters. You may use that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

   A. $1/(9\pi)$ centimeters per minute

   B. $1/(4\pi)$ centimeters per minute

   C. $1/(3\pi)$ centimeters per minute

   D. $1/9$ centimeters per minute

   E. $1/4$ centimeters per minute

3. Find the absolute minimum value of $f(x) = |x+1| + 2|x-2|$ on the interval $[-3, 3]$.

   A. -1

   B. 1

   C. 3

   D. 5

   E. 7

4. Suppose f is a differentiable function on the real line, $f(2) = 3$ and $4 \leq f'(x) \leq 6$. Which interval is equal to the set of all possible values for $f(4)$?

   A. $[-5, 15]$

   B. $[7, 9]$

   C. $[10, 12]$

   D. $[11, 13]$

   E. $[11, 15]$

5. On which of the following intervals does the minimum value of the function $f(x) = x^2 - 4x + 3$ occur at an endpoint?

   A. $[0, 2]$

   B. $[0, 3]$

   C. $[0, 4]$

   D. $[1, 3]$

   E. $[1, 4]$

6. Let $f(x) = \dfrac{e^x}{x} - 3$. Which of the statements is true?

   A. f is increasing on $(-\infty, \infty)$

   B. f is increasing on $(-1, 1)$

   C. f is increasing on $(0, 1)$

   D. f is decreasing on $(0, 1)$

   E. f is decreasing on $(1, \infty)$

7. Which condition can guarantee that the function $g(x) = \dfrac{1}{x^2 - 1}$ is concave up?

   A. $x^2 - 1$ is negative

   B. $x^2 - 1$ is positive

   C. $3x^2 - 1$ is negative

   D. $3x^2 - 1$ is positive

   E. $3x^2 + 1$ is positive

8. How many inflection points does the function $h(\theta) = \theta + \cos\theta$ have on the interval $[0, 2\pi]$?

   A. 0

   B. 1

   C. 2

   D. 3

   E. 4

9. Find the value of $\lim_{x\to 0} \dfrac{x^{16} + 11\sin(x)}{2021\ln(1+x)}$.

   A. $11/2021$

   B. $16/2021$

   C. $27/2021$

   D. $11/16$

   E. None of the above

10. Evaluate $\lim_{x\to\infty} x\sin(1/x)$.

   A. -1

   B. 0

   C. 1

   D. ∞

   E. None of the above

11. Find the largest area of a rectangle if its perimeter is 20 m.

A. 10 square meters

B. 16 square meters

C. 36 square meters

D. 25 square meters

E. 100 square meters

12. Find an antiderivative of $f(x) = 1/x + 11\sin(x) + 16\cos(x)$ on $(0, \infty)$.

A. $-1/x^2 - 11\cos(x) + 16\sin(x) + C$

B. $1/x^2 + 11\cos(x) + 16\sin(x) + C$

C. $\ln(x) + 11\cos(x) + 16\sin(x) + C$

D. $\ln(x) - 11\cos(x) + 16\sin(x) + C$

E. $\ln(x) + 11\cos(x) - 16\sin(x) + C$

13. Evaluate the left sum L_4 for $f(x) = 9 - x^2$, $-2 \leq x \leq 2$. In other words find the Riemann sum with four equal-length subintervals, taking the sample points to be the left endpoints.

    A. 26

    B. 30

    C. $30\frac{2}{3}$

    D. 31

    E. 34

14. Use the fact that $\displaystyle\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$ to find

$$\lim_{N\to\infty} \frac{2}{N} \sum_{k=1}^{N} \left(\frac{4k}{N} + 3 \right).$$

    A. 6

    B. 7

    C. 10

    D. 11

    E. 14

15. If $\int_2^8 f(x)\,dx = 17$ and $\int_2^4 f(x)\,dx = 6$, find $\int_4^8 2f(x)\,dx$.

A. -22

B. -11

C. 11

D. 22

E. 46

16. If $\int_1^5 f(x)\,dx = 8$ and $\int_1^5 g(x)\,dx = 3$, find $\int_1^5 (5f(x) - 7g(x) + 2)\,dx$

A. -33

B. 7

C. 13

D. 21

E. 27

Free Response Questions
Show all of your work

17. Merckx road runs north-south and Indurain road runs east-west. Eddy is bicycling south on Merckx road at a speed of 30 kilometers/hour and Miguel is riding east on Indurain road at a speed of 35 kilometers/hour.

    (a) At 12 noon today Eddy is 60 kilometers north the intersection Merckx and Indurain Roads and Miguel is 80 kilometers east of the intersection. What is the distance between these cyclists at 12 noon?

    (b) What is the rate of change of the distance between the cyclists at 12 noon?

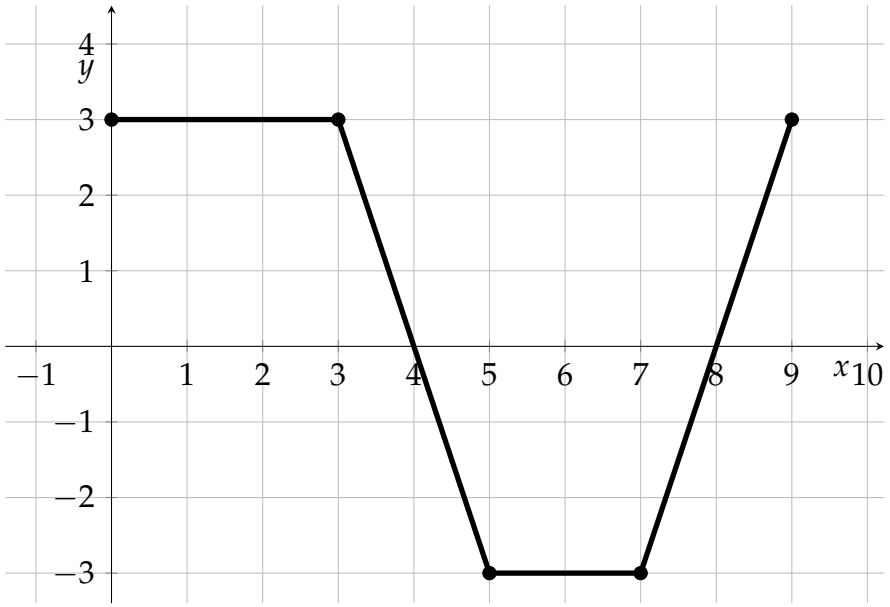
18. Let $f(x) = x^4 - 8x^2$ for all real numbers x. Using calculus answer the following questions.

(a) At what values of x does f have a local minimum?

(b) What are the x-coordinate(s) of the inflection point(s)?

(c) On what interval(s) is the function concave up?

19. Using calculus, find the point on the curve $y = \sqrt{2x}$ that is closest to the point $(3, 0)$ for $x \geq 0$.



20. The graph of f is shown above. Evaluate each integral by interpreting it in terms of areas.

(a) $\int_3^0 f(x)\,dx$

(b) $\int_5^7 f(x)\,dx$

(c) $\int_5^9 |f(x)|\,dx$