Name: ______________________________ Section and/or TA: __________________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **11** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **12** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) The area of circle is increasing at a rate of 12π square units per minute. Find the rate of change (in units/minute) of the radius when the radius is 3 units.

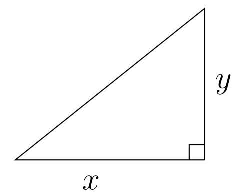
   A. 3 B. 2 C. 1/2 D. 1/3 E. 1/6

2. (5 points) Suppose y is the solution of the initial value problem $\begin{cases} y' = 2y \\ y(0) = 3. \end{cases}$ Find the function y.

   A. $y(t) = e^{3(t-2)}$
   B. $y(t) = 3e^{2t}$
   C. $y(t) = 3e^{t-2}$
   D. $y(t) = 2e^{t-3}$
   E. $y(t) = 2e^{3t}$

3. (5 points)

   In the right triangle pictured at right, the lengths of the two legs, x and y, sum to 8. What is the area of the largest possible area for a triangle that satisfies this condition?



   A. 8 B. $8\sqrt{2}$ C. 16 D. 2 E. 4

4. (5 points) The derivative of a function f is $f'(x) = x(x-1)e^x$. Select the correct statement.

   A. The function f does not have a local maximum or a local minimum at 0.

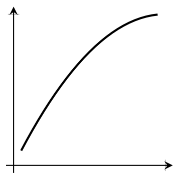
   B. The point $x = 0$ is a singular point for f.

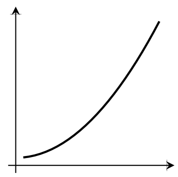
   C. The point $x = 0$ is not a critical point for f.

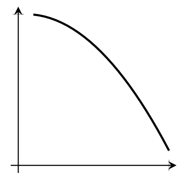
   D. The function f has a local minimum at 0.

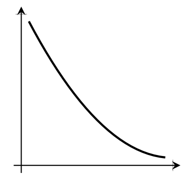
   E. The function f has a local maximum at 0.

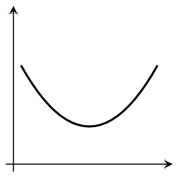
5. (5 points) Select the graph of the function which is increasing and concave down.

A. 

B. 

C. 

D. 

E. 

6. (5 points) Find the point c so that $f(c)$ is the global minimum value of $f(x) = 2x - x^2$ on the interval $[0, 4]$

A. 0 B. 4 C. 3 D. 2 E. 1

7. (5 points) Let $f(x) = xe^{-2x}$. Find the value c so that $(c, f(c))$ is an inflection point for f.

   A. -1 B. $1/2$ C. 0 D. 1 E. $-1/2$

8. (5 points) Suppose that the derivative of a function f is $f'(x) = x(x-1)$. Find the interval or intervals where f is decreasing.

   A. $[1, \infty)$
   B. $[0, 1]$
   C. $(-\infty, 0]$ and $[1, \infty)$
   D. $(-\infty, 0]$
   E. $(-\infty, 1]$

9. (5 points) Let $f(x) = \arctan(x)$. Find the critical point(s) of f.
   - A. $1, -1$
   - B. 1
   - C. 0
   - D. There are no critical points
   - E. -1

10. (5 points) Find the value K for which the limit $\displaystyle\lim_{x\to\pi/2} \frac{\sin(x) - K}{x - \pi/2}$ exists and has a finite value.
    - A. -1
    - B. 1
    - C. 0
    - D. $-\sqrt{2}/2$
    - E. $\sqrt{2}/2$

11. (5 points) Find the limit

$$\lim_{x \to 0} \frac{1 - \cos(x^2)}{x \sin(x)}.$$

A. 2 B. 1 C. -1 D. 0 E. -2

12. (5 points) The function F is an anti-derivative of x^2 and $F(1) = 1$. Find F.

A. $F(x) = 2x + 1$

B. $F(x) = \dfrac{x^3}{3} + \dfrac{2}{3}$

C. $F(x) = \dfrac{x^3}{3} - \dfrac{2}{3}$

D. $F(x) = 2x - 1$

E. $F(x) = \dfrac{x^3}{3} + 1$

Free response questions, show all work

13. (10 points) A ladder of length 5 meters is leaning against a wall. The top of the ladder is sliding down the wall at a rate of 0.2 meters/second. You should assume that the wall is perpendicular to the ground.

    (a) Draw a sketch summarizing the information in the problem.

    (b) At the time when the bottom of the ladder is 4 meters away from the wall, find the speed with which the bottom of the ladder is moving.

    (c) At the time when the bottom of the ladder is 4 meters away from the wall, is the bottom of the ladder moving towards the wall or away from the wall?

14. (10 points) Suppose that the mass of a radioactive substance X decays exponentially with a half-life of 6 years.

    (a) If we have 100 grams of substance X at time $t = 0$, find A and k so that the function $m(t) = Ae^{-kt}$ gives the mass of substance X after t years. Give exact values, rather than decimal approximations for A and k.

    (b) After how many years will we have 50 grams of substance X remaining? Your answer should be rounded correctly to 3 decimal places or more accurate.

    (c) After how many years will we have 10 grams of substance X remaining? Your answer should be rounded correctly to 3 decimal places or more accurate.

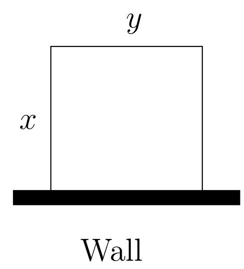
15. (10 points)

We want to build a rectangular pen. One side of the pen will be part of long, straight rock wall and the other three sides are to be constructed using 60 meters of fencing.

(a) Express the area as a function of the sidelength x in the sketch at right and give the domain of this function.

(b) Use calculus to find the dimensions and the area of the pen with the largest possible area.

(c) Explain why you have found the pen with the largest possible area.



16. (10 points) A function F satisfies $F''(x) = e^x + 6x$, $F(0) = 0$ and $F'(0) = 1$.

   (a) Find F'.

   (b) Find F.