MA 113 Calculus I   Spring 2014
Exam 3   Tuesday, 15 April 2014

Name: ───────────────────

Section: ─────────────────

Last 4 digits of student ID #: ───────

This exam has ten multiple choice questions
(five points each) and five free response ques-
tions (ten points each). Additional blank sheets
are available if necessary for scratch work. No
books or notes may be used. Turn off your
cell phones and do not wear ear-plugs during
the exam. You may use a calculator, but not
one which has symbolic manipulation capa-
bilities.

On the multiple choice problems:

- Select your answer by placing an X in
  the appropriate square of the multiple
  choice answer box on the front page of
  the exam.

- Carefully check your answers. No credit
  will be given for answers other than those
  indicated on the *multiple choice answer
  box*.

On the free response problems:

- Clearly indicate your answer and the
  reasoning used to arrive at that answer
  *(unsupported answers may not receive
  credit),*

- Give exact answers, rather than deci-
  mal approximations to the answer (un-
  less otherwise stated).

Each free response question is followed by
space to write your answer. Please write your
solutions neatly in the space below the ques-
tion.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Let $f(x) = x^3 - 2x^2$. Find the largest open interval on which f is decreasing.

   (A) $(0, 4/3)$

   (B) $(0, 2)$

   (C) $(-\infty, 2/3)$

   (D) $(2/3, \infty)$

   (E) $(-2, 0)$

2. If $f'(1) = 0$ and $f''(1) > 0$, then which of the following is false

   (A) f has a critical point at 1.

   (B) f has a local minimum at 1

   (C) f has a local maximum at 1

   (D) f is differentiable at 1

   (E) f is continuous at 1

3. Give the linear approximation to $f(x) = \sqrt{3x + 3}$ at $x = 2$.

(A) $L(x) = \dfrac{1}{2}x + 2$

(B) $L(x) = \dfrac{1}{2}x$

(C) $L(x) = \dfrac{1}{2}x + \dfrac{1}{2}$

(D) $L(x) = \dfrac{3}{2}x$

(E) $L(x) = \dfrac{3}{2}x - \dfrac{5}{2}$

4. Let $f(x) = \sin(2x)$. On which subintervals of $[0, \pi]$ is f concave down?

(A) $(0, \pi/4), (3\pi/4, \pi)$

(B) $(\pi/4, 3\pi/4)$

(C) $(0, \pi/2)$

(D) $(\pi/2, \pi)$

(E) $(0, \pi)$

5. Let $f(x) = x^3 + 3x$. Which of the following statements is true?

    (A) f has no local extrema

    (B) f has a local maximum at -1

    (C) f has a local maximum at 1

    (D) f has a local minimum at 1

    (E) f has a local minimum at -1

6. Let $f(x) = \dfrac{(2x+1)(x-2)(1-x)}{x^3}$. and find $\lim\limits_{x\to\infty} f(x)$.

    (A) 0

    (B) 2

    (C) -2

    (D) ∞

    (E) $-\infty$

7. If $\displaystyle\sum_{k=1}^{n} a_k = n^2 + n$, find $\displaystyle\sum_{k=11}^{20} a_k$.

    (A) 42

    (B) 110

    (C) 132

    (D) 310

    (E) 420

8. Let $f(x) = x^3 - 2$. Use Newton's method to find a solution of $f(x) = 0$ beginning with $x_0 = 2$. Give a decimal approximation of x_2, correctly rounded to three decimal places.

    (A) -2.444

    (B) 1.208

    (C) 1.260

    (D) 1.296

    (E) 1.500

9. Let $f(x) = x^3$ on the interval $[-1, 1) = \{x : -1 \leq x < 1\}$. Find the absolute maximum and minimum values of f on the interval $[-1, 1)$.

   (A) The absolute minimum value is 1 and the absolute maximum value is -1.

   (B) The absolute minimum value is -1 and the absolute maximum value is 1.

   (C) The absolute minimum value is -1 and the absolute maximum value is 0.

   (D) The absolute minimum value is -1 and there is no absolute maximum value.

   (E) There is no absolute minimum value and the absolute maximum value is 0.

10. Let $f(x) = x^2$. Divide the interval $[0,2]$ into three subintervals of equal length and compute R_3, the 3rd right-endpoint approximation to the area of the region $R = \{(x, y) : 0 \leq x \leq 2, \ 0 \leq y \leq x^2\}$.

   (A) $40/27$

   (B) $56/27$

   (C) $112/27$

   (D) $22/9$

   (E) $8/3$

11. (a) State the mean value theorem.

    (b) For each function and interval determine if the mean value theorem applies. If the theorem does apply, state this. If the theorem does not apply, explain which hypothesis fails.

        i. $f(x) = x\sin(x)$ on the interval $[2, 42]$.
   ii. $g(x) = |x|$ on the interval $[-1, 1]$.

12. Let $f(x) = e^{2x}$.

    (a) Find $L(x)$, the linearization of $f(x)$ at 0. Put your answer in the form $L(x) = mx + b$.

    (b) Find
    $$\lim_{x \to 0} \frac{f(x) - L(x)}{x^2}.$$

13. We have a rectangular piece of cardboard of area 200 cm^2. A box with no top is to be constructed by removing squares of side length 2 cm from each corner and folding up the remaining flaps.

    (a) If the rectangle is a cm $\times$ b cm, find a function $V(a)$ which gives the volume of the box as a function of a. For which values of a is it possible to construct a box?

    (b) Find the dimensions a and b which give the box of largest volume and explain how you know you have found the largest volume.
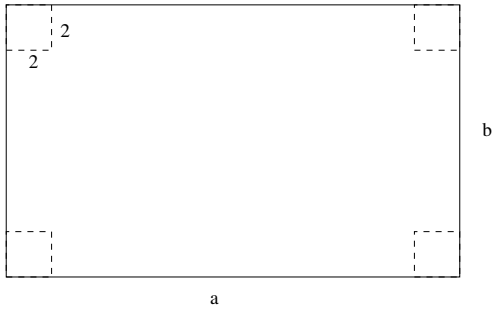
14. Suppose that a particle moves so that at time t seconds, its acceleration is $a(t) = 6t - 2$ cm/second^2. The position at time $t = 0$ is 7 cm to the right of the origin and the velocity at time $t = 1$ is 2 cm/second.

   (a) Find a function which gives the position at all times t.

   (b) Find the velocity at $t = 2$.

15. You may find one or more of the following formulæ useful for this problem.

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}, \qquad \sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}.$$

Consider the sum

$$S_N = \sum_{k=1}^{N} (2 + \frac{3k}{N}).$$

(a) Find a closed from expression for S_N.

(b) Find the limit $\lim_{N \to \infty} \frac{1}{N} S_N$.