MA 113 Calculus I — Spring 2015

Exam 3 — Tuesday, 14 April 2015

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

D,A,B,C,E   D,B,A,A,E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Suppose $f(x) = x^3 + 6x^2 - 15x + 7$. How many critical points does f have in the interval $(-3, 4)$?

   (A) 4

   (B) 3

   (C) 2

   (D) 1

   (E) 0

2. Suppose $f(x) = x^4 + 2x^3 - 36x^2 + 17x + 11$. On which of the following intervals is the graph of $f(x)$ concave up?

   (A) $(-\infty, -3)$, $(2, \infty)$

   (B) $(-\infty, -2)$, $(3, \infty)$

   (C) $(-2, 3)$

   (D) $(-3, 2)$

   (E) None of the above

3. Suppose that x and y are chosen such that $2x + y = 8$ and that xy is as large as possible. Then $x - y$ equals

   (A) -4

   (B) -2

   (C) 0

   (D) 2

   (E) 4

Record the correct answer to the following problems on the front page of this exam.

4. Suppose that $f(x)$ is a differentiable function with $3 \leq f'(x) \leq 5$ for all x in the interval $(2,4)$. If $f(2) = 8$, we can use the Mean Value Theorem for $f(x)$ on the interval $[2,4]$ to determine that the largest possible value of $f(4)$ is

   (A) 14

   (B) 16

   (C) 18

   (D) 20

   (E) 22

5. The indefinite integral $\int(\sqrt{x} + e^{4x} - 7\sin(3x))dx$ equals

   (A) $\dfrac{x^{\frac{3}{2}}}{3} + e^{4x} + \dfrac{7\cos(3x)}{3} + C$

   (B) $\dfrac{2x^{\frac{3}{2}}}{3} + e^{4x} - \dfrac{7\cos(3x)}{3} + C$

   (C) $\dfrac{2x^{\frac{3}{2}}}{3} + e^{4x} + \dfrac{7\cos(3x)}{3} + C$

   (D) $\dfrac{2x^{\frac{3}{2}}}{3} + \dfrac{e^{4x}}{4} - \dfrac{7\cos(3x)}{3} + C$

   (E) $\dfrac{2x^{\frac{3}{2}}}{3} + \dfrac{e^{4x}}{4} + \dfrac{7\cos(3x)}{3} + C$

Record the correct answer to the following problems on the front page of this exam.

6. Suppose that $\sum_{j=1}^{n} a_j = 3$, $\sum_{j=1}^{n} b_j = 2$, and $\sum_{j=1}^{n} c_j = 4$. Then $\sum_{j=1}^{n}(3a_j + 2b_j - 4c_j)$ equals

   (A) -6
   (B) -5
   (C) -4
   (D) -3
   (E) -2

7. Let $f(x) = x^3 - 2x - 1$. If we use Newton's Method to solve $f(x) = 0$ with starting point $x_0 = 3$, find x_1.

   (A) 2.15
   (B) 2.20
   (C) 2.25
   (D) 2.30
   (E) 2.35

Record the correct answer to the following problems on the front page of this exam.

8. Suppose f has a continuous first derivative in $[0,3]$ and that $f'(x) > 0$ for $x < 2$, and $f'(x) < 0$ for $x > 2$. Which of the following statements is true?

   (A) f has a local maximum at $x = 2$.

   (B) f' is discontinuous at $x = 2$.

   (C) f has a local maximum at $x = 0$ and $x = 3$.

   (D) $f(x) > 0$ for $x \in [0,3]$, $x \neq 2$.

   (E) The tangent line to f at $x = 2$ is vertical.

9. Given that f has a continuous second derivative in $[-3,3]$ with $f''(x) > 0$ for $x \in (-3,0)$, $f''(x) < 0$ for $x \in (0,1)$, and $f''(x) > 0$ for $x \in (1,3)$, which of the following statements is false?

   (A) Any critical point in $(0,1)$ is a local min.

   (B) f' is increasing in $(-3,0)$.

   (C) The tangent lines of f in $(1,3)$ lie below the graph of f.

   (D) f has an inflection point at $x = 1$.

   (E) f' is decreasing for $x \in (0,1)$.

10. Which of the following is the antiderivative of $\dfrac{x\cos(x)+3}{x}$?

   (A) $\dfrac{-x^2\sin x - 3}{x^2} + C$

   (B) $\cos(x) + 3\ln|x| + C$

   (C) $\cos x + \dfrac{3}{x^2} + C$

   (D) $\sin x - \dfrac{3}{x^2} + C$

   (E) $\sin x + 3\ln|x| + C$

Free Response Questions: Show your work!

11. Show your work in both parts below. Using a calculator to guess the limit may not receive full credit.

(a) Find $\lim_{x \to 0} \dfrac{\cos(x) - 1}{e^x - x - 1}$.

By L'Hopital's rule, $\lim_{x \to 0} \dfrac{\cos(x) - 1}{e^x - x - 1} = \lim_{x \to 0} \dfrac{-\sin(x)}{e^x - 1} = \lim_{x \to 0} \dfrac{-\cos(x)}{e^x} = -1.$

(b) Find $\lim_{x \to \infty} \dfrac{2x^3}{(3x-1)(6x+1)(4x+2)}$.

$\lim_{x \to \infty} \dfrac{2x^3}{(3x-1)(6x+1)(4x+2)} = \lim_{x \to \infty} 2 \cdot \dfrac{x}{3x-1} \cdot \dfrac{x}{6x+1} \cdot \dfrac{x}{4x+2}$
$= 2 \cdot \dfrac{1}{3} \cdot \dfrac{1}{6} \cdot \dfrac{1}{4} = \dfrac{1}{36}.$

12. A cylindrical can has volume $V = 120$ cm^3, radius r and height h. Find r and h that minimize the surface area S of the can. (Recall that $V = \pi r^2 h$ and $S = 2\pi r^2 + 2\pi rh$.)

    $120 = \pi r^2 h$. $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r(\frac{120}{\pi r^2}) = 2\pi r^2 + \frac{240}{r}$.
    $S'(r) = 4\pi r - \frac{240}{r^2} = 0$. $4\pi r^3 = 240$. $r^3 = \frac{60}{\pi}$. This gives a minimum value because $S''(r) = 4\pi + \frac{480}{r^3} > 0$.
    $h = \frac{120}{\pi r^2} = \frac{120}{\pi} \cdot (\frac{\pi}{60} r) = 2r$ because $\frac{1}{r^2} = \frac{\pi}{60} r$.

Free Response Questions: Show your work!

13. A man of height 2 meters walks away from a 5-meter lamppost at a speed of 1 m/s.

    (a) Draw a picture with appropriate parts labeled to illustrate this problem.

    Let x denote the distance of the man from the lamppost. Let y denote the length of his shadow.

    (b) Find the rate at which the man's shadow is increasing in length. (Suggestion: Use similar triangles to find a relation between the variables.)

    Note that $\frac{dx}{dt} = 1$. Then $\frac{x+y}{5} = \frac{y}{2}$ by similar triangles.
    This gives $2x + 2y = 5y$, $2x = 3y$.
    Thus $2\frac{dx}{dt} = 3\frac{dy}{dt}$, $\frac{dy}{dt} = \frac{2}{3}\frac{dx}{dt} = \frac{2}{3}$.

14. An object moves along a straight line so that its acceleration is $2t - 1$ cm/second^2. The position at time $t = 0$ is 5 cm to the right of the origin and the velocity at time $t = 2$ is 3 cm/second.

    (a) Find a function that gives the velocity at all times t.
    $v(t) = t^2 - t + C_1$. Since $3 = v(2) = 2^2 - 2 + C_1$, we have $C_1 = 1$. Thus $v(t) = t^2 - t + 1$.

    (b) Find a function that gives the position at all times t.
    $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + t + C_2$. Since $s(0) = 5$, we have $C_2 = 5$. Thus $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + t + 5$.

Free Response Questions: Show your work!

15. You may use the following formulas in this problem.

$$\sum_{j=1}^{N} j = \frac{N(N+1)}{2}, \quad \sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$$

We want to find the area A under the graph of $f(x) = 4x^2 + 1$ and above the x-axis over the interval $[1,3]$. We let R_N denote the right endpoint approximation for this area when we divide the interval $[1,3]$ into N equal subintervals.

(a) Compute R_4.
$\Delta x = \frac{3-1}{4} = \frac{1}{2}$. Then
$R_4 = \frac{1}{2}(f(1.5) + f(2) + f(2.5) + f(3)) = \frac{1}{2}(10 + 17 + 26 + 37) = \frac{1}{2}(90) = 45.$

(b) For an arbitrary positive integer N one can show that

$$R_N = \sum_{j=1}^{N} \left(\frac{10}{N} + \frac{32}{N^2} j + \frac{32}{N^3} j^2 \right).$$

Using this result, find the area A by computing $\lim_{N\to\infty} R_N$.

$$\begin{aligned}
\lim_{N\to\infty} R_N &= \lim_{N\to\infty} \left(\frac{10}{N} \cdot N + \frac{32}{N^2} \sum_{j=1}^{N} j + \frac{32}{N^3} \sum_{j=1}^{N} j^2 \right) \\
&= \lim_{N\to\infty} \left(\frac{10}{N} \cdot N + \frac{32}{N^2} \cdot \frac{N(N+1)}{2} + \frac{32}{N^3} \cdot \frac{N(N+1)(2N+1)}{6} \right) \\
&= \lim_{N\to\infty} \left(10 + 16 \cdot \frac{N}{N} \cdot \frac{N+1}{N} + \frac{32}{6} \cdot \frac{N}{N} \cdot \frac{N+1}{N} \cdot \frac{2N+1}{N} \right) \\
&= 10 + 16 + \frac{32}{6} \cdot 2 = 26 + \frac{32}{3} = \frac{110}{3}.
\end{aligned}$$