MA 113 Calculus I — Spring 2017

Exam 3 — Tuesday, April 11, 2017

**Name:** ______________________________

**Section:** ____________________________

**Last 4 digits of student ID #:** _______

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the true/false and multiple choice problems:**

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

**On the free response problems:**

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

| **True/False** | | |
|---|---|---|
| 1 | T | F |
| 2 | T | F |
| 3 | T | F |
| 4 | T | F |
| 5 | T | F |

| **Multiple Choice** | | | | | |
|---|---|---|---|---|---|
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |
| 13 | A | B | C | D | E |
| 14 | A | B | C | D | E |
| 15 | A | B | C | D | E |

**Overall Exam Scores**

| Question | Score | Total |
|---|---|---|
| TF | | 10 |
| MC | | 50 |
| 16 | | 10 |
| 17 | | 10 |
| 18 | | 10 |
| 19 | | 10 |
| Total | | 100 |

## Free Response Questions: Show your work!

16. (a) State the Mean Value Theorem.

See textbook.

(b) Find all values $c$ that satisfy the conditions of the mean value theorem for

$$f(x) = \sin(x) - \cos(x)$$

on the interval $[a, b] = [2\pi, 4\pi]$.

We need to solve $f'(x) = \dfrac{f(b)-f(a)}{b-a} = \dfrac{\sin(4\pi) - \cos(4\pi) - \sin(2\pi) + \cos(2\pi)}{4\pi - 2\pi}$

$= 0.$

Since $f'(x) = \cos x + \sin x$, we need all solns to $\cos x + \sin x = 0$ on $(2\pi, 4\pi)$.

Since $\sin x = -\cos x$ for $x = 2\pi + \frac{3}{4}\pi,\ 4\pi - \frac{1}{4}\pi$,

These are the solutions.

## Free Response Questions: Show your work!

17. (a) Evaluate the following limit. Be sure to explain your reasoning.

$$\lim_{x\to-\infty} x^2e^x = \lim_{x\to-\infty} \frac{x^2}{e^{-x}} \quad \text{of form } \frac{\infty}{\infty}.$$

By L'Hopital, we get $\lim_{x\to-\infty} \frac{2x}{-e^{-x}} \underset{\substack{\uparrow \\ \text{L'H} \\ \text{again}}}{=} \lim_{x\to-\infty} \frac{2}{e^{-x}} = 0.$

(b) Find all values of $A$ for which we can use L'Hopital's rule to evaluate

$$\lim_{x\to3} \frac{x^2+Ax-3}{x-3}.$$

Then use L'Hopital's rule to evaluate the limit.

Since denominator is 0 at $x=3$, need numerator $=0$ at $x=3$.

$3^2+3A-3=0 \Rightarrow 3A=-6 \Rightarrow A=-2$. This is the only value that allows L'Hopital.

$$\lim_{x\to3} \frac{x^2-2x-3}{x-3} \underset{\substack{\uparrow \\ \text{L'H}}}{=} \lim_{x\to3} \frac{2x-2}{1} = 4.$$

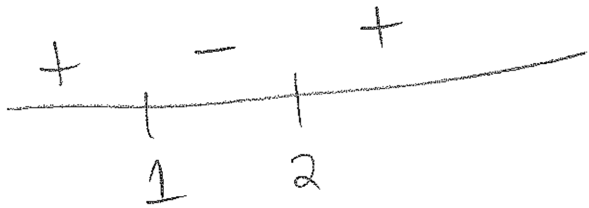
## Free Response Questions: Show your work!

18. Consider the function $f(x) = x^3/3 - (3/2)x^2 + 2x - 2$. Use methods of Calculus to solve the following. Be sure to show your work and explain how you obtained your answers.

    (a) Find the interval(s) where the function $f(x)$ is increasing and the interval(s) where the function $f(x)$ is decreasing.

Since $f'(x)$ is a polynomial, it is defined everywhere.

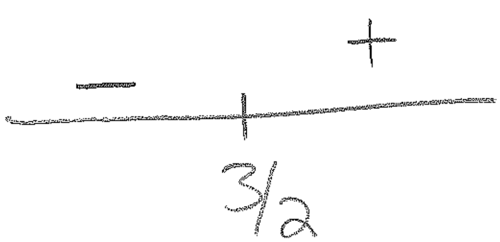
$f'(x) = x^2 - 3x + 2 = (x-2)(x-1) = 0 \Rightarrow x = 1, 2.$




$f(x)$ is increasing on $(-\infty, 1) \cup (2, \infty)$ and decreasing on $(1, 2)$.

    (b) Find the interval(s) where the graph of $f(x)$ is concave up and the interval(s) where the graph of $f(x)$ is concave down.

$f''(x) = 2x - 3 \Rightarrow f''(x) = 0$ at $x = \frac{3}{2}$.



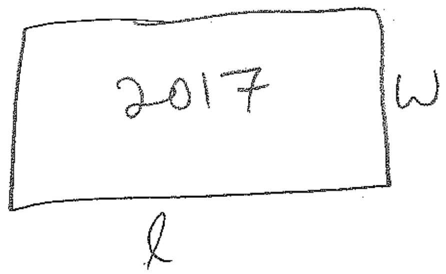

$f(x)$ is concave down on $(-\infty, \frac{3}{2})$ and concave up on $(\frac{3}{2}, \infty)$.

## Free Response Questions: Show your work!

Name: ______________________ Student ID Number: ________

19. In this problem you will determine the minimum possible perimeter for a rectangle whose area is 2017.

(a) Draw a picture of the rectangle and label/name all quantities.




$l \cdot w = 2017$

(b) Write the perimeter as a function of one variable, and state the domain of the perimeter function that agrees with the physical constraints of the problem.

$P = 2l + 2w = 2l + 2 \cdot \frac{2017}{l}$. Domain is $(0, \infty)$ since we need only $l > 0$.

(c) Use methods of Calculus to determine what the dimensions of the rectangle are that provide minimal perimeter length.

$P'(l) = 2 - \frac{2 \cdot 2017}{l^2}$. This is defined for all $l > 0$,
and $P'(l) = 0$ when $l = \sqrt{2017}$. Since $P'(l) < 0$
for $l < \sqrt{2017}$, and $P'(l) > 0$ for $l > \sqrt{2017}$, we have at
$l = \sqrt{2017}$ P has an abs. minimum. The dimensions
are $l = \sqrt{2017} = w$.