MA 113 Calculus I — Spring 2019

Exam ~~2 ALT~~ 3 — Tuesday, ~~8 March~~ 9 Apr 2019

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question						
1	A	B	C	D	E	B
2	A	B	C	D	E	D
3	A	B	C	D	E	A
4	A	B	C	D	E	D
5	A	B	C	D	E	C
6	A	B	C	D	E	A
7	A	B	C	D	E	A
8	A	B	C	D	E	B
9	A	B	C	D	E	C
10	A	B	C	D	E	E
11	A	B	C	D	E	B
12	A	B	C	D	E	A

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

* KEY * KEY * KEY * KEY *

Free Response Questions: Show your work!

(5 pts) 15. (a) Find the value of $\lim_{x\to 0} \frac{\sin 6x}{\tan 7x}$.

"$\frac{0}{0}$" Use L'Hopital's Rule! (+1) don't need to write this if they've attempted L'Hopital's

$$\lim_{x\to 0} \frac{\sin(6x)}{\tan(7x)} = \lim_{x\to 0} \frac{6\cos(6x)}{7\sec^2(7x)} = \frac{6\cos(0)}{7\sec^2(0)} = \frac{6\cdot 1}{7\cdot 1} = \boxed{\frac{6}{7}}$$

(+1) derivative of top

(+1) derivative of bottom

(+1) plug in 0

(+1) final answer

(5 pts) (b) If $f''(x) = 3x + 1$ where $f(0) = 2$ and $f(1) = 2$, find $f(x)$.

$$f'(x) = \frac{3x^2}{2} + \frac{1x}{1} + C = \underline{\frac{3}{2}x^2 + x + C}$$ (+1) antiderivative

$$f(x) = \frac{1}{3}\cdot\frac{3}{2}x^3 + \frac{1x^2}{2} + Cx + d = \underline{\frac{1}{2}x^3 + \frac{1}{2}x^2 + Cx + d}$$ (+1) antiderivative

$$f(0) = \frac{1}{2}(0)^3 + \frac{1}{2}(0)^2 + C(0) + d = \underline{d = 2}$$ (+1) solve for "d"

$$f(1) = \frac{1}{2}(1)^3 + \frac{1}{2}(1)^2 + C(1) + d = 1 + C + d = C + 3 = 2$$

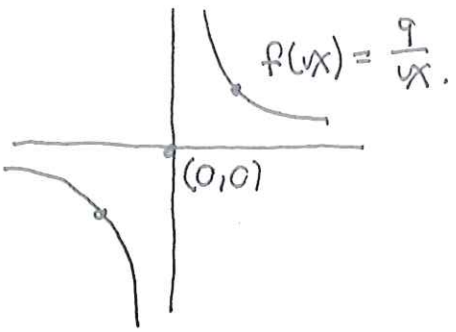
$$\underline{C = -1}$$ (+1) solve for "C"

$$\boxed{f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 1x + 2}$$ (+1) final answer

Team: Reddy, Plougher (Daniel), Ashby (Chase)

Free Response Questions: Show your work!

14. Find the point(s) on the hyperbola $y = \frac{9}{x}$ that is (are) closest to $(0, 0)$. (Hint: minimize the square of an appropriate distance function.)



Minimize

$$D = (x-0)^2 + (y-0)^2 = x^2 + y^2, \quad 2$$

subject to (x, y) is on the hyperbola, that is,

$$y = \frac{9}{x}.$$

Minimize (change to a function of one variable.)

$$D = x^2 + y^2 = x^2 + \left(\frac{9}{x}\right)^2 = x^2 + \frac{81}{x^2} \quad \Big\} \; 2$$

Differentiate

$$\frac{dD}{dx} = 2x - \frac{81}{x^3} \cdot 2 \quad 1.$$

Critical points:

$\frac{dD}{dx} = 0$

Set derivative = zero.

$$2x - \frac{81(2)}{x^3} = 0 \quad 1.$$

$$2x = \frac{81}{x^3}(2)$$

$$x^4 = 81 = 3^4 \; (= (-3)^4)$$

$$\Rightarrow x = 3, -3. \quad 1.$$

$\frac{dD}{dx}$ undefined when $x = 0$. However, $x = 0$ is not in the domain of $f(x) = \frac{9}{x}$.

2 pts for arguing absolute min in some manner.

$\searrow \; \nearrow \; \searrow \; \nearrow$

$f'(x)$	$- - -$ 0 $+++$		$- - -$ 0 $+++$	
x	-3	0	3	

Test pt

$f'(-10) = -20 - \frac{81(2)}{-1000} = \ominus$

$f'(-1) = -2 + \frac{81(2)}{1} = \oplus$

$f'(1) = 2 - 81(2) = \ominus$

$f'(10) = 20 - \frac{81(2)}{1000} = \oplus$

Thus, min. distance is @ $(3, 9/3)$ & $(-3, -9/3)$. $\Big\} \; 1$.

Free Response Questions: Show your work!

13. Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

(a) Find the value of $\sum_{i=1}^{12}(5i+2)$. $= 5 \cdot \sum_{i=1}^{12} i + 2\sum_{i=1}^{12} 1$

$= 5 \cdot \frac{12 \cdot 13}{2} + 2 \cdot 12$

$= 5 \cdot 6 \cdot 13 + 24$

$= 414$

+1 for attempting problem.

-1 Arithmetic error

-1 linearity of sum

-1 plug into formula

-1 adding 2 incorrectly

(b) Calculate L_3 for $f(x) = 2 + x^2$ over the interval $[0, 3]$.

$\Delta x = \frac{3-0}{3} = 1$. $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, so

$L_3 = \sum_{i=0}^{2} f(x_i) \cdot \Delta x = (2+0^2) \cdot 1 + (2+1^2) \cdot 1 + (2+2^2) \cdot 1$

$= 2 + 3 + 6$

$= 11.$

+1 for attempting problem

-1 for using R_3

-1 for using wrong $f(x)$

-1 for adding all 4 endpts.

Free Response Questions: Show your work!

16. This problem concerns the definition of the derivative using limits.

4 pts (a) State the Mean Value Theorem. One point per line.

If f is continuous on $[a,b]$ +1 pt

and differentiable on (a,b) +1pt

then there exists $c \in (a,b)$ such that +1pt

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$ +1 pt Equivalent expressions ok.

6 pts (b) Suppose that $g(x)$ is differentiable for all x and that $-2 \leq g'(x) \leq 4$ for all x. Also assume that $g(1) = 3$. Find the largest possible value for $g(3)$.

+1 for any reasonable attempt.

Set up:

$$-2 \leq \frac{g(3)-g(1)}{3-1} \leq 4$$ +2 pts for set up: 1pt for bounds, 1pt for $\frac{g(3)-g(1)}{3-1}$

Note: Using any non-Calculus way to solve this correctly gives 3/6

$$\Rightarrow -2 \leq \frac{g(3)-3}{2} \leq 4$$ +1 showing work

$$\Rightarrow -4 \leq g(3)-3 \leq 8$$

$$\Rightarrow -1 \leq g(3) \leq 11$$ +1 completely correct work

The largest possible value for $g(3)$ is 11. +1 stating largest value.