Exam 3

Form A

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 14 multiple choice questions and 3 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**8**	A	B	C	D	E
2	A	B	C	D	E	**9**	A	B	C	D	E
3	A	B	C	D	E	**10**	A	B	C	D	E
4	A	B	C	D	E	**11**	A	B	C	D	E
5	A	B	C	D	E	**12**	A	B	C	D	E
6	A	B	C	D	E	**13**	A	B	C	D	E
7	A	B	C	D	E	**14**	A	B	C	D	E

SCORE

Multiple Choice	15	16	17	Total Score
70	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. If $\int_0^9 f(x)\,dx = 6$ and $\int_0^9 g(x)\,dx = 5$, find $\int_0^9 (5f(x) - 7g(x) + 2)\,dx$

   A. -5

   B. 13

   C. 65

   D. 67

   E. 83

2. Find the general antiderivative of $f(x) = 1/x + \sin(x) + 2\cos(x)$ on $(0, \infty)$.

   A. $-1/x^2 - \cos(x) + 2\sin(x) + C$

   B. $1/x^2 + \cos(x) + 2\sin(x) + C$

   C. $\ln(x) + \cos(x) + 2\sin(x) + C$

   D. $\ln(x) - \cos(x) + 2\sin(x) + C$

   E. $\ln(x) + \cos(x) - 2\sin(x) + C$

3. Find the largest area of a rectangle if its perimeter is 60 meters.

   A. 15 square meters

   B. 32 square meters

   C. 50 square meters

   D. 225 square meters

   E. 900 square meters

4. Suppose f is a differentiable function, $f(2) = 3$ and $f'(x) \leq 6$ for $2 \leq x \leq 4$, how large can $f(4)$ possibly be?

   A. 6

   B. 9

   C. 12

   D. 13

   E. 15

5. Find all of the critical numbers for the function $g(x) = x^3 - 2x^2 - 4x + 144$.

   A. $x = 0$ only

   B. $x = 2$ only

   C. $x = \pm 12$

   D. $x = -\frac{2}{3}$ and $x = 2$

   E. $x = 3$ and $x = 2$

6. Find the value of the limit

$$\lim_{x \to 0} \frac{3\sin(4x) - 12x}{x^3}.$$

   A. -2

   B. -4

   C. -8

   D. -16

   E. -32

7. If $\int_0^3 f(x)dx = 13$ and $\int_0^2 f(x)dx = 7$, find $\int_2^3 f(x)dx$.

A. -6

B. 6

C. 7

D. 13

E. 20

8. Find $f(x)$ if $f'(x) = 3x^2 - 2\sin(x)$ and $f(0) = 5$.

A. $f(x) = x^3 + 2\cos(x)$

B. $f(x) = 6x - 2\sin(x) + 5$

C. $f(x) = x^3 + 2\cos(x) + 3$

D. $f(x) = x^3 + 2\cos(x) - 5$

E. $f(x) = x^3 - 2\cos(x) + 7$

9. Where does the function $f(x) = x^3 - 9x^2$ have a point of inflection?

   A. $x = -4$

   B. $x = 0$

   C. $x = 1$

   D. $x = 2$

   E. $x = 3$

10. An athlete runs with velocity 24 km/h for 10 minutes, 18 km/h for 5 minutes , and 30 km/h for 5 minutes. Compute the total distance traveled.

    A. 5 km

    B. 6 km

    C. 7 km

    D. 8 km

    E. 9 km

11. Find the absolute maximum value of $f(x) = x^3 - 6x^2 + 9x - 5$ on the interval $[0, 5]$.

   A. 10
   B. 14
   C. 15
   D. 20
   E. 48

12. Find $\int_0^4 f(x)\,dx$ if

$$f(x) = \begin{cases} 2 & x < 2 \\ -2x + 1 & x \geq 2 \end{cases}$$

   A. -12
   B. -6
   C. 0
   D. 8
   E. 22

13. You are given that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

    A. Local maximum value of $f(x)$ at $x = 0$ and local minimum values of $f(x)$ at $x = -2, 4$.

    B. Local maximum value of $f(x)$ at $x = 2$ and local minimum values of $f(x)$ at $x = -2, 4$.

    C. Local maximum values of $f(x)$ at $x = -2, 4$ and local minimum value of $f(x)$ at $x = 0$.

    D. Local maximum values of $f(x)$ at $x = -2, 2$ and local minimum values of $f(x)$ at $x = 0, 4$.

    E. Local maximum values of $f(x)$ at $x = 0, 4$ and local minimum values of $f(x)$ at $x = -2, 2$.

14. Assume that $g''(x) = x^2(x-2)(x-4)$. Find the points of inflection of the function $g(x)$. (Read the problem carefully. The given function is $g''(x)$, not $g(x)$.)

    A. $x = 2$

    B. $x = 4$

    C. $x = 0, 4$

    D. $x = 2, 4$

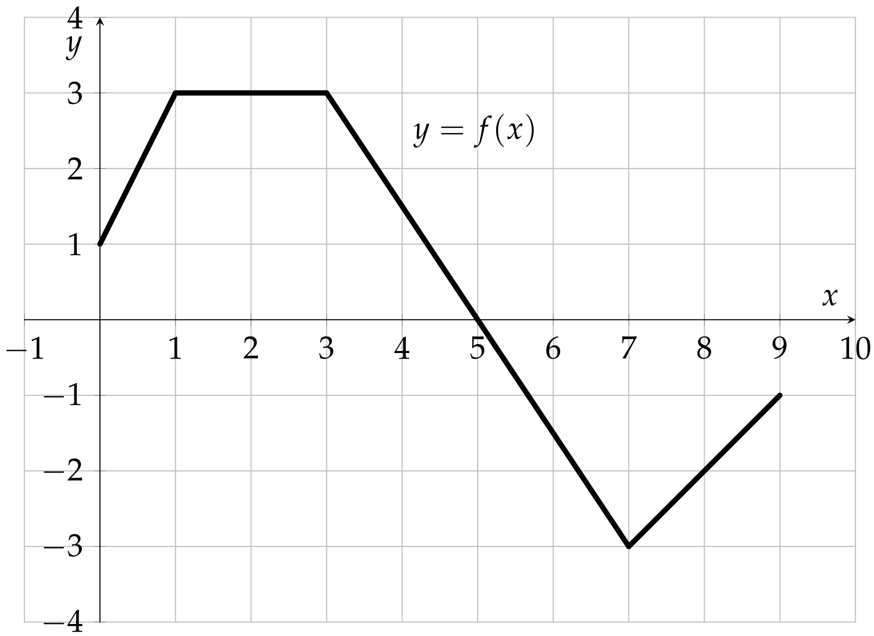
    E. $x = 0, 2, 4$

Free Response Questions
Show all of your work

15. (a) Find the following limit:

$$\lim_{x \to 0} \frac{1 - e^x}{\ln(x+1)}$$

(b) Find the value of A for which we can use l'Hôpital's rule to evaluate the limit $\lim_{x \to 2} \frac{x^2 + Ax - 2}{x - 2}$ and find the value of the limit.



16. A graph of $f(x)$ is shown above. Using the geometry of the graph, evaluate the definite integrals. The grid lines in the above graph are one unit apart.

(a) $\int_0^2 f(x)\,dx$

(b) $\int_0^5 f(x)\,dx$

(c) $\int_5^7 f(x)\,dx$

(d) $\int_3^7 f(x)\,dx$

(e) $\int_0^9 f(x)\,dx$

17. Let $f(x) = x^4 - 32x^2 + 7$. Be sure to justify each of your answers below.

   (a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

   (b) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

   (c) Find the points that give local maximum values of $f(x)$, the points that give local minimum values of $f(x)$, and the points of inflection of $f(x)$.