MA 113 Calculus I Fall 2013
Exam 4 Wednesday, 18 December 2013

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the linearization, $L(x)$, of the function $f(x) = \frac{3x+1}{2x-1}$ at $x = 3$.

   (A) $L(x) = -5(x-3) + 2$

   (B) $L(x) = 2(x-3) + 2$

   (C) $L(x) = \dfrac{1}{5}(x-2) + 3$

   (D) $L(x) = -\dfrac{1}{5}(x-3) + 2$

   (E) $L(x) = \dfrac{1}{5}(x-3) + 2$

2. Consider the curve given by the equation $y^2 + xe^{2y} = 2$. Which of the following is the slope of the tangent line at the point $(2, 0)$.

   (A) 0

   (B) The curve does not have a tangent line at $(2, 0)$.

   (C) $-\dfrac{1}{4}$

   (D) $\dfrac{1}{4}$

   (E) 2

Record the correct answer to the following problems on the front page of this exam.

3. Let $c > 0$. Consider the function $f(x) = 8x^2 + c\ln(x)$ on the interval $(0, \infty)$. Which of the following is a point of inflection of f?

   (A) $\left(\sqrt{c},\, f(\sqrt{c})\right)$

   (B) $\left(\frac{\sqrt{c}}{4},\, f(\frac{\sqrt{c}}{4})\right)$

   (C) $\left(\frac{c}{4},\, f(\frac{c}{4})\right)$

   (D) $\left(\frac{\sqrt{c}}{2},\, f(\frac{\sqrt{c}}{2})\right)$

   (E) f does not have a point of inflection.

4. Let $g(x)$ be a differentiable function such that $\lim_{x\to 0} g(x) = 1$ and $\lim_{x\to 0} g'(x) = 4$. Compute the limit

$$\lim_{x\to 0} \frac{x\cos(x)}{g(x) - 1}.$$

   (A) 0

   (B) $\frac{1}{4}$

   (C) $\frac{1}{3}$

   (D) $\frac{1}{2}$

   (E) ∞

Record the correct answer to the following problems on the front page of this exam.

5. Let f be a differentiable function on $\mathbb{R} = (-\infty, \infty)$, and

$$f(2) = 6, \;\; f'(2) = 5.$$

Let $h(x) = xf(x^2 - 2)$. Compute $h'(2)$.

(A) 26

(B) 36

(C) 46

(D) 56

(E) 66

6. Determine the constant b such that $\displaystyle\int_1^{e^2} \frac{b}{x}\,dx = 8$.

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Record the correct answer to the following problems on the front page of this exam.

7. A particle is traveling along a straight line with a velocity of

$$v(t) = 3t^2 - 6t \quad \text{meters/minute.}$$

Compute the particle's total distance traveled over the time interval $[1, 4]$.

(A) 16 meters

(B) 18 meters

(C) 20 meters

(D) 22 meters

(E) 24 meters

8. Let b be a constant. Consider the function $f(x) = e^{x^2-2bx-2}$. Find the maximal open interval on which f is decreasing.

(A) (b, ∞)

(B) $(-b, \infty)$

(C) $(-b, b)$

(D) $(-\infty, -b)$

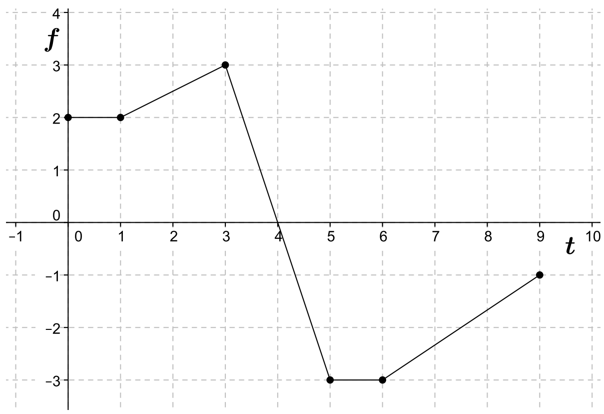
(E) $(-\infty, b)$

Record the correct answer to the following problems on the front page of this exam.

Let

$$F(x) = \int_0^x f(t)dt \quad \text{for } x \text{ in the interval } [0, 9],$$

where $f(t)$ is the function with the graph shown in the following picture. This function F will be used for Problems 9 and 10 on this page.



9. Determine $F(3)$.

   (A) 6

   (B) 7

   (C) 8

   (D) 9

   (E) 10

10. Which of the following statements is true?

    (A) F is decreasing on the interval $[3, 5]$.

    (B) F has a local maximum at 4.

    (C) F has a local minimum at 4.

    (D) F is increasing on the interval $[6, 9]$.

    (E) $F(5) = F(6)$.

Free Response Questions: Show your work!

11. Evaluate the following integrals.

    (a) $\displaystyle\int \frac{2e^x}{(5e^x+3)^4}\,dx.$

    (b) $\displaystyle\int_1^x \frac{4+t\sin(t)}{2t}\,dt$, where $x>0$.

Free Response Questions: Show your work!

12. Let $B(t)$ denote the number of bacteria at time t (measured in hours) in a certain culture. The population grows exponentially, thus

$$B(t) = Ce^{kt} \text{ for some positive constants } C \text{ and } k.$$

Suppose the population has a doubling time of 20 hours.

(a) Find the constant k. Give the exact answer.

(b) How long does it take for the population to increase to seven times the initial population? Give the exact answer and a decimal approximation accurate to two decimal places.

(c) If at time $t = 1$ there are 800 bacteria, how many were there at time $t = 0$? Give the exact answer and a decimal approximation accurate to two decimal places.

Free Response Questions: Show your work!

13. (a) State both parts of the Fundamental Theorem of Calculus. Use complete sentences and make sure to include all assumptions.

(b) Find the derivative of the function g defined as $g(x) = \displaystyle\int_2^{x^2-2} t\ln(t^2)\,dt$.

Free Response Questions: Show your work!

14. Let u and v be positive real numbers such that $uv = 4$. Find the minimum value of $u^3 + 12v$. Determine the values of u and v for which the minimum is attained. As always, justify your answers!

Free Response Questions: Show your work!

15. Consider the two functions

$$f(x) = x^3 - 7x, \quad g(x) = 2x.$$

(a) Find all points of intersection of the graphs of f and g.

(b) Compute the area of the region enclosed by the graphs of f and g. Present also a sketch of the two graphs and the enclosed region.