MA 113 Calculus I Fall 2015
Exam 4 Tuesday, 15 December 2015

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-phones during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

1. The definite integral:

$$\int_0^{\sqrt{\frac{\pi}{2}}} x\cos(x^2)\,dx$$

is equal to

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{3}{2}$
(E) $\frac{\pi}{2}$

2. A population of bacteria grows exponentially. The initial population is 1000 cells and after one hour the population is 2000 cells. What is the population after 5 hours?

(A) 4,000 cells
(B) 6,000 cells
(C) 8,000 cells
(D) 12,000 cells
(E) 32,000 cells

Record the correct answer to the following problems on the front page of this exam.

3. If $f(x) = \tan(x)$ and $g(x) = 2x^2 + x$, the derivative

$$\frac{d}{dx} f(g(x))$$

is equal to

(A) $\sec(4x + 1)$
(B) $\sec^2(2x^2 + 1)$
(C) $(4x + 1)\sec^2(2x^2 + x)$
(D) $\csc^2(2x^2 + x)$
(E) $(4x + 1)\csc^2(2x^2 + x)$

4. The definite integral:

$$\int_0^4 \left(3x^{\frac{1}{2}} - 2xe^{-x^2}\right)\, dx.$$

is equal to

(A) $10 + e^{-16}$
(B) $15 + e^{-16}$
(C) $16 + e^{-16}$
(D) $16 - e^{-16}$
(E) $16 - 2e^{-16}$

Record the correct answer to the following problems on the front page of this exam.

5. A function f is continuous on $[1, 4]$ and differentiable on $(1, 4)$, and assumes the values $f(2) = 1$ and $f(3) = 4$. According to the Mean Value Theorem, there is a point c satisfying $2 < c < 3$ at which the slope of the tangent line to the graph of f at the point c is equal to

   (A) 1

   (B) 2

   (C) 3

   (D) 4

   (E) 5

6. The critical points of $g(x) = 2x^3 - 3x^2 - 12x$ are

   (A) $\{1, 2\}$

   (B) $\{-2, 2\}$

   (C) $\{-1, 2\}$

   (D) $\{-1, -2\}$

   (E) $\{1, -2\}$

Record the correct answer to the following problems on the front page of this exam.

7. Find the slope of the tangent line to the graph of the function $A(x)$ at $x = \dfrac{\pi}{3}$. The function $A(x)$ is defined by

$$A(x) = \int_0^x \cos^2(s)\, ds.$$

The slope is

(A) 0

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{3}{4}$

(E) 1

8. The following limit:

$$\lim_{x \to 1} \frac{x^3 + 4x^2 - 3x - 2}{x^2 + 17x - 18}$$

is equal to

(A) $\dfrac{8}{19}$

(B) $\dfrac{1}{2}$

(C) 1

(D) $\dfrac{3}{2}$

(E) Does not exist.

Record the correct answer to the following problems on the front page of this exam.

9. Suppose $y(x)$ satisfies the differential equation

$$y'(x) = 7y(x),$$

with the initial condition $y(0) = 15$. The solution $y(x)$ is

(A) $y(x) = 7e^{15x}$
(B) $y(x) = 105e^{7x}$
(C) $y(x) = 15e^{7x}$
(D) $y(x) = 15e^{-7x}$
(E) $y(x) = 15e^{15x}$

10. The derivative of $f(x) = x\ln(x^2)$, for $x > 0$, is

(A) $2(\ln(x) + 1)$
(B) $2x^2\ln(x)$
(C) $x\ln(x) + 1$
(D) $\dfrac{2}{x}$
(E) $2\ln(x)$

11. Compute the area between the two curves $f(x) = x^2 + 2$ and $g(x) = 2x + 5$. Sketch the graphs and show all your work.

12. Consider the function $h(x) = x^4 - 8x^2 + 16$ on the interval $[-4, 3]$.

    (a) Find the critical points of h.

    (b) Find the absolute and local extrema of h and where they occur on the interval $[-4, 3]$. Classify all of these points as local or absolute maxima or local or absolute minima. Use calculus to justify your answers.

13. The velocity of a particle moving on a straight line is

$$v(t) = 3t^2 - 24t + 36 \; meters/sec,$$

for $t \in [0, 6]$.

(a) Find the time intervals in $[0, 6]$ on which the particle is moving backwards, and on which the particle is moving forward.

(b) Find the displacement of the particle over the time interval $[0, 6]$.

(c) Find the total distance traveled by the particle over the time interval $[0, 6]$.

14. Find the pair of positive numbers (x, y) satisfying $4x + y = 9$ that maximizes the function $M(x, y) = x^2y$.

15. Two cars start from an intersection at the same time. Car 1 travels north on a straight road at 20 mph and car 2 travels east on a straight road at 40 mph. Find the rate at which the distance between them is increasing after two hours.