MA 113 Calculus I — Fall 2017
Exam 4 — Monday, 11 December 2017

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear earbuds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. Find the equation of the tangent line to the graph of $f(x) = \tan(x)$ at the point $x = \frac{\pi}{6}$.

   (A) $y = \sqrt{3} + \dfrac{4}{3}\left(x - \dfrac{\pi}{6}\right)$

   (B) $y = \dfrac{\sqrt{3}}{3} + \dfrac{4}{3}\left(x - \dfrac{\pi}{6}\right)$

   (C) $y = \dfrac{\sqrt{3}}{3} + \dfrac{3}{4}\left(x - \dfrac{\pi}{6}\right)$

   (D) $y = \sqrt{3} + \dfrac{3}{4}\left(x - \dfrac{\pi}{6}\right)$

   (E) None of the above

2. Assume that $\dfrac{dy}{dx} = ky$ for some real number k. Also assume that $y(0) = 7$ and that $y(4) = 12$. Find k.

   (A) $k = \frac{1}{4} \cdot \frac{12}{7}$

   (B) $k = 4\ln(\frac{12}{7})$

   (C) $k = \frac{12}{7}\ln(4)$

   (D) $k = \frac{1}{4}\ln(\frac{12}{7})$

   (E) $k = \frac{1}{4}\ln(12)$

Record the correct answer to the following problems on the front page of this exam.

3. Suppose that $f'(x) = x^3 + 3x^2 - 24x + 5$. Find the interval or intervals where $f'(x)$ is increasing.

   (A) $(-\infty, 2) \cup (4, \infty)$

   (B) $(-\infty, 2)$

   (C) $(-\infty, -4) \cup (2, \infty)$

   (D) $(-4, \infty)$

   (E) $(-4, 2)$

4. Let $f(x) = e^{\sqrt{x}} + \dfrac{\cos(x)}{x}$. Find $f'(x)$.

   (A) $\dfrac{1}{2}\sqrt{x}e^{\sqrt{x}} - \left(\dfrac{x\sin(x) - \cos(x)}{x^2}\right)$

   (B) $\dfrac{1}{2}\sqrt{x}e^{\sqrt{x}} - \left(\dfrac{x\sin(x) + \cos(x)}{x^2}\right)$

   (C) $\dfrac{1}{2}\sqrt{x}e^{\sqrt{x}} + \left(\dfrac{x\sin(x) - \cos(x)}{x^2}\right)$

   (D) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}} - \left(\dfrac{x\sin(x) - \cos(x)}{x^2}\right)$

   (E) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}} - \left(\dfrac{x\sin(x) + \cos(x)}{x^2}\right)$

Record the correct answer to the following problems on the front page of this exam.

5. Let $f(x) = \sin(x)\ln(x^2 + 4)$. Find $f'(x)$.

   (A) $\dfrac{\sin(x)}{x^2 + 4} - \cos(x)\ln(x^2 + 4)$

   (B) $\dfrac{\sin(x)}{x^2 + 4} + \cos(x)\ln(x^2 + 4)$

   (C) $\sin(x)\frac{2x}{x^2+4} - \cos(x)\ln(x^2 + 4)$

   (D) $\sin(x)\frac{2x}{x^2+4} + \cos(x)\ln(x^2 + 4)$

   (E) None of the above

6. Assume that $f(x)$ is a differentiable function at all real numbers. Suppose that $x = 1, 3, 5, 7, 9$ are values where f has a local maximum value or a local minimum value. Which of the following statements is true?

   (A) f has exactly 5 critical points.

   (B) f has at most 5 critical points.

   (C) f has at least 5 critical points.

   (D) At least three of the values give a local maximum value for f.

   (E) At least three of the values give a local minimum value for f.

Record the correct answer to the following problems on the front page of this exam.

7. Assume that $f(x)$ is a function where $f'(x)$ and $f''(x)$ are defined at all real numbers. Suppose that the following information holds.

$$f'(1) = f'(4) = f'(7) = f'(10) = 0.$$

$$f''(1) = 9, \;\; f''(4) = -8, \;\; f''(7) = 3, \;\; f''(10) = 0.$$

Which of the following statements is true?

(A) Local maximum value of f at $x = 4$ and local minimum values of f at $x = 1, 7$

(B) Local maximum values of f at $x = 4, 10$ and local minimum values of f at $x = 1, 7$

(C) Local maximum value of f at $x = 4$ and local minimum values of f at $x = 1, 7, 10$

(D) Local maximum values of f at $x = 1, 7$ and local minimum value of f at $x = 4$

(E) Local maximum values of f at $x = 1, 7$ and local minimum values of f at $x = 4, 10$

8. Find the linearization of $f(x) = \sqrt[3]{1+x}$ at $x = 0$.

(A) $L(x) = \sqrt[3]{1+x}$

(B) $L(x) = 1 + \frac{1}{3}x$

(C) $L(x) = 1 - \frac{1}{3}x$

(D) $L(x) = 1 + \sqrt[3]{x}$

(E) None of the above

Record the correct answer to the following problems on the front page of this exam.

9. The length of a rectangle is increasing at a rate of 6 cm/s and the width is increasing at a rate of 4 cm/s. When the length is 16 cm and the width is 12 cm, how fast is the area of the rectangle increasing?

   (A) 112 cm^2/s

   (B) 192 cm^2/s

   (C) 24 cm^2/s

   (D) 144 cm^2/s

   (E) 136 cm^2/s

10. Assume that $y = x^3 + x^2$ and $x = g(t)$ where g is a differentiable function. Suppose that $\dfrac{dy}{dt} = 60$ when $x = 2$. Find $g'(t)$ when $x = 2$.

    (A) 2.75

    (B) 3

    (C) 3.25

    (D) 3.5

    (E) 3.75

Record the correct answer to the following problems on the front page of this exam.

11. Let $f(x) = \int_2^{x^2} e^{4t}\, dt$. Find $f\,'(2)$.

    (A) $4e^4$

    (B) $16e^4$

    (C) $4e^{16}$

    (D) $16e^{16}$

    (E) $8e^4$

12. Compute $\displaystyle\int_1^9 \left(2t + \sqrt{t} + \frac{1}{t^2}\right) dt$.

    (A) 98

    (B) $98\frac{1}{9}$

    (C) $98\frac{2}{9}$

    (D) $98\frac{4}{9}$

    (E) $98\frac{5}{9}$

13. A box with a square base and an open top is to have a volume of 18 m^3. Material for the base costs \$16 per square meter and material for the sides costs \$12 per square meter. Find the dimensions of the cheapest box. Make sure to justify that you have minimized the cost.

Free Response Questions: Show your work!

14. (a) Find $\int x^3\sqrt{x^4+4}\,dx$.

(b) Find $\int \frac{x^2}{x^3+6}\,dx$.

15. (a) Find the 3^{rd} degree Taylor polynomial $T_3(x)$ for the function $f(x) = \ln(1+x)$ at $a = 0$. (Recall that

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n(n-1)\cdots 3\cdot 2}(x-a)^n.)$$

(b) Use $T_3(x)$ above to estimate $\ln(\frac{1}{2})$.
(Show your work. No credit given for a calculator estimate.)

Free Response Questions: Show your work!

16. A particle moves along a straight line so that its velocity at time t seconds is

$$v(t) = 3t^2 + 6t - 24 = 3(t-2)(t+4)$$

meters per second.

(a) Find the displacement of the particle during the time interval $1 \leq t \leq 4$.

(b) Find the total distance traveled during the time interval $1 \leq t \leq 4$.