Name: ______________________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	(A)	(B)	(C)	(D)	(E)	**7**	(A)	(B)	(C)	(D)	(E)
2	(A)	(B)	(C)	(D)	(E)	**8**	(A)	(B)	(C)	(D)	(E)
3	(A)	(B)	(C)	(D)	(E)	**9**	(A)	(B)	(C)	(D)	(E)
4	(A)	(B)	(C)	(D)	(E)	**10**	(A)	(B)	(C)	(D)	(E)
5	(A)	(B)	(C)	(D)	(E)	**11**	(A)	(B)	(C)	(D)	(E)
6	(A)	(B)	(C)	(D)	(E)	**12**	(A)	(B)	(C)	(D)	(E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let f be defined by

$$f(x) = \begin{cases} \dfrac{\sin(x)(1+\cos(x))}{x}, & x \neq 0 \\ p, & x = 0 \end{cases}$$

Find p so that f is continuous everywhere.

A. 1

B. 2

C. -1

D. 0

E. There is no value of p for which f is continuous

Solution: WW 1.6 #7,8, WW2.8 #6 or WW 3.7 #2.

2. (5 points) Suppose $0 \leq f(x) \leq x^2 - 2x + 1$. There is one value of a for which we can use the squeeze theorem to show $\lim_{x \to a} f(x)$ exists. What is this value of a?

A. 1

B. 0

C. -2

D. 2

E. -1

Solution: WW 1.4, #10

3. (5 points) Select the correct description of the limit

$$\lim_{h\to 0} \frac{\sin(\pi+h)-\sin(\pi)}{h}$$

A. The derivative $f'(h)$ of $f(x)=\sin(x)$

B. The derivative $f'(\pi)$ of $f(x)=\sin(x)$

C. The derivative $f'(\pi)$ of $f(x)=\cos(x)$

D. The derivative $f'(0)$ of $f(x)=\cos(x)$

E. The derivative $f'(0)$ of $f(x)=\sin(x)$

Solution:

4. (5 points) Consider the curve defined by $x^2-xy+3y^2=5$. Find the tangent line to this curve at the point $(x,y)=(2,1)$.

A. $y-1=-\frac{1}{2}(x-2)$

B. $y-1=\frac{1}{2}(x-2)$

C. $y-1=-\frac{3}{4}(x-2)$

D. $y-1=\frac{3}{4}(x-2)$

E. $y-1=\frac{5}{4}(x-2)$

Solution: Compare WS 2.11 #1, #4.

5. (5 points) At time $t = 0$, an object is thrown upwards from the top of a building that is 100 meters above the ground. The object is thrown with a velocity of 12 meters/second. Find its height above the ground after 4 seconds. Use that the acceleration due to gravity is 10 meters/second^2 downwards.

   A. 28 meters

   B. 58 meters

   C. 68 meters

   D. 38 meters

   E. 48 meters

Solution: WS 3.1

6. (5 points) The *derivative* of f is $f'(x) = x(x-2)(x-3)$. Find the interval or intervals where f is decreasing.

   A. $(-\infty, 0)$ *and* $(2, 3)$

   B. $(-\infty, 0)$ and $(3, \infty)$

   C. $(0, 2)$ and $(3, \infty)$

   D. $(3, \infty)$

   E. $(0, 2)$

Solution: WW 3.6.1-2, #1

7. (5 points) Let $f(x) = 1/x$. Find the left Riemann sum approximation for $\int_2^4 f(x)\,dx$ which uses three subintervals of equal length. On each subinterval, use the value of f at the left endpoint of the subinterval for the height of the rectangle. Round your answer to three decimal places.

A. 0.779 *B. 0.783* C. 0.781 D. 0.777 E. 0.775

Solution: Compare WW II-1.1 #8

The division points are $\{2, 8/3, 10/3, 12/3\}$. The Riemann sum is

$$\frac{2}{3}\left(\frac{1}{2} + \frac{3}{8} + \frac{3}{10}\right) = \frac{47}{60} \approx 0.783.$$

8. (5 points) Consider the limit $\lim_{n\to\infty} \frac{6}{n} \sum_{k=1}^{n} \sin(3 + \frac{2k}{n})$. Express the limit as an integral.

A. $\int_3^6 \sin(2x)\,dx$

B. $\int_3^6 6\sin(x)\,dx$

C. $\int_3^5 6\sin(x)\,dx$

D. $\int_3^6 2\sin(x)\,dx$

E. $\int_3^5 3\sin(x)\,dx$

Solution: WW II-1.1 #6

9. (5 points) Suppose $\int_{100}^{300} f(x)\,dx = 1$ and $\int_{200}^{300} f(x)\,dx = 2$. Find $\int_{100}^{200} 2f(x)\,dx$.

   A. 2

   B. -4

   C. -1

   D. 1

   E. -2

Solution: WS II-1.2, #1.

10. (5 points) Let $f(x) = -\sqrt{16 - x^2}$. Evaluate the integral $\int_0^4 f(x)\,dx$ by using the formula for the area of a circle.

   A. 16π

   B. 8π

   C. 4π

   D. -16π

   E. -4π

Solution: WW II-1.2 #2

11. (5 points) Evaluate the indefinite integral $\int \frac{x}{1+x^2}\,dx$

A. $\arctan(x) + C$

B. $\frac{1}{2}\arctan(x) + C$

C. $\frac{1}{2}\ln(1+x^2) + C$

D. $2\arctan(x) + C$

E. $\ln(1+x^2) + C$

Solution: Compare WW II-1.4 #8

12. (5 points) Evaluate the integral $\int_0^a e^{x/a}\,dx$. Assume $a \neq 0$.

A. $\frac{1}{a}(e-1)$

B. $e - 1$

C. $a(e^{1/a} - 1)$

D. $a(e-1)$

E. $e^{1/a} - 1$

Solution: WW II-1.4

Free response questions, show all work

13. (10 points) Compute the derivatives.

(a) $\frac{d}{dx}(x^3e^{2x^2})$

(b) $\frac{d}{dx}\left(\frac{3+\sin(x)}{3-\sin(x)}\right)$

(c) $\frac{d}{dt}\sqrt{t^2+3}$

Solution: a) Use the product rule and chain rule to find

$$\frac{d}{dx}(x^3e^{2x^2}) = 3x^2e^{2x^2} + x^3 \cdot 4xe^{2x^2} = (3x^2+4x^4)e^{2x^2}$$

b) Use the quotient rule to write

$$\frac{d}{dx}(\frac{3+\sin(x)}{3-\sin(x)}) = \frac{\cos(x)(3-\sin(x)) - (-\cos(x))(3+\sin(x))}{(3-\sin(x))^2} = \frac{6\cos(x)}{(3-\sin(x))^2}.$$

Alternate solution: Write $\frac{3+\sin(x)}{3-\sin(x)} = [(3+\sin(x))(3-\sin(x))^{-1}]$. Differentiating with the product rule and chain rule gives

$$\frac{d}{dx}(3+\sin(x))(3-\sin(x))^{-1}$$
$$= \cos(x)(3-\sin(x))^{-1} + (3+\sin(x))(-\cos(x))(-1)(3-\sin(x))^{-2}$$
$$= \cos(x)(3-\sin(x))^{-1} + (3+\sin(x))\cos(x)(3-\sin(x))^{-2}$$

c) We write the radical as a power and use the chain rule to find

$$\frac{d}{dx}(t^2+3)^{1/2} = \frac{1}{2}2t(t^2+3)^{-1/2} = \frac{t}{\sqrt{t^2+3}}.$$

Grading:

a) Product rule (2 point), chain rule (1 point), answer (1 point).

b) Quotient rule (2 points), answer (1 point)

c) Derivative of square root (1 point), chain rule (1 point), answer (1 point)

Notes: Problem does not ask for simplification. A student who applies a rule, but with some minor error may receive the credit for the rule, but not for the correct answer.

14. (10 points) A 4 meter ladder leans against a wall. When the bottom of the ladder is 1 meter away from the wall, the bottom of the ladder is sliding away from the wall at 0.3 meter/second.

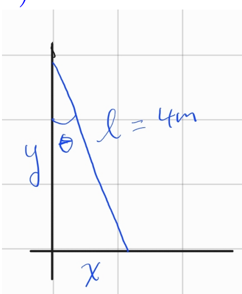
    (a) Make a sketch summarizing the information in the problem. Label the quantities you use in your solution.

    (b) Find the height of the top of the ladder when the base of the ladder is 1 meter away from the wall.

    (c) Find the rate of change of the *angle* between the ladder and the wall when the base of the ladder is 1 meter from the wall. Is the angle increasing or decreasing?

Solution:

a)



b) Using Pythagoras's theorem, we have $\ell^2 = x^2 + y^2$. When $x = 1$ and $\ell = 4$, we can solve to find $y = \sqrt{4^2 - 1^2} = \sqrt{15}$ meters.

c) We have $\theta = \arcsin(x/\ell) = \arcsin(x/4)$. Differentiating we have

$$\theta' = \frac{1}{\sqrt{1-(x/4)^2}} \cdot \frac{x'}{4}$$

When $x = 1$ meter and $x' = 0.3$ meters/second, we have $\theta' = \frac{0.3}{4}\frac{1}{\sqrt{1-(1/16)}} = \frac{3}{10\sqrt{15}}$ seconds^{-1}.

The angle is increasing since the derivative is positive.

Alternate solution: We write $\sin(\theta) = x/4$ and differentiate with respect to time to find $\theta' \cos(\theta) = x'/4$ or $\theta' = \frac{x'}{\cos(\theta)4} = \frac{x'}{\sqrt{15}} = \frac{0.3}{\sqrt{15}}$ since $\cos(\theta) = y/4 = \sqrt{15}/4$ when $x = 1$.

Grading:

a) Sketch with at least one side or angle labeled (1 point)

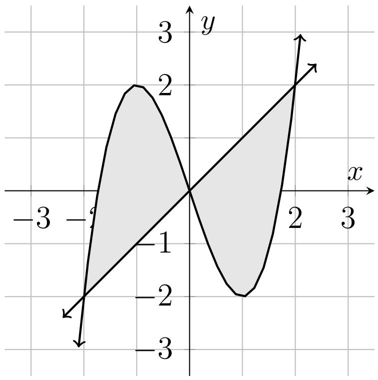
b) Use of Pythagoras (1 point), answer (1 point)

c) Equation involving θ and two sides (1 point). Differentiate to find equation involving θ' (2 points), answer for θ' (2 points), angle is increasing (1 point). Units in part b) and c) (1 point).

Note: If we begin with $\tan(\theta) = y/x$ or $\cos(\theta) = y/4$, it is more challenging to find the answer since we need to find y'.

WW 3.2 #7

15. (10 points) Below are the graphs of $f(x) = x^3 - 3x$ and $g(x) = x$. Find the area of the shaded region.



Solution: WS II-§1.5 #4.

The graphs of $x^3 - 3x$ and x intersect for x where $x = x^3 - 3x$ or $x^3 - 4x = x(x^2 - 4) = 0$. The solutions are $x = -2, 0, 2$.

The area between the graphs of $y = x$ and $y = x^3 - 4x$ is

$$\int_{-2}^{2} |x^3 - 4x| \, dx.$$

We use that $x^3 - 3x \geq x$ for $-2 \leq x \leq 0$ and $x \geq x^3 - x$ for $0 \leq x \leq 2$ to write the integrand without absolute values and obtain that the area is

$$\int_{-2}^{0} (x^3 - 4x) \, dx + \int_{0}^{2} (4x - x^3) \, dx.$$

We use FTC II to evaluate the integrals,

$$\begin{aligned} \int_{-2}^{0} (x^3 - 4x) \, dx + \int_{0}^{2} (4x - x^3) \, dx &= \left(\frac{x^4}{4} - 2x^2\right)\Big|_{x=-2}^{0} + \left(2x^2 - \frac{x^4}{4}\right)\Big|_{x=0}^{2} \\ &= 0 - \left(\frac{16}{4} - 8\right) + \left(\frac{16}{4} - 8\right) - 0 = 8. \end{aligned}$$

Grading:

Points of intersection (2 points) (may read from graph), expression of area as an integral (1 point), expressing area without absolute values (3 points), finding antiderivative (2 points), value for area (2 points).

Students may read points of intersection off of graph. Students who go directly to an expression without absolute values should receive 6 points for the expression.

16. (10 points) Let $f(x) = \int_0^x e^{-t^2}\, dt$.

(a) Find f' and f''.

(b) Find the intervals where f is increasing and decreasing.

(c) Find the intervals where f is concave up and concave down.

(d) Find the inflection point(s), if any.

Solution: WW II-1.3 #10

a) According to FTC I, the derivative of f, $f'(x) = e^{-x^2}$. Then using the chain rule, we have $f''(x) = -2xe^{-x^2}$.

b) Since $f'(x) > 0$ for all x, the function f is increasing on $(-\infty, +\infty)$.

c) Since $f''(x) > 0$ for x in $(-\infty, 0)$, the function f is concave up there. Since $f''(x) < 0$ for x in $(0, \infty)$, the function f is concave down on $(0, \infty)$.

d) The function changes concavity at $x = 0$, so $(x, y) = (0, 0)$ is an inflection point.

Grading:

a) f' and use of FTC I (3 points), f'' (1 point)

b) Interval of increase (2 points)

c) Intervals of concavity (2 points)

d) Inflection point (2 points), only require x-coordinate for full credit.