Name: ______________________________ Section and/or TA: ____________________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Find the value of the limit $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$.

   A. $-1/2$ B. $1/2$ C. $-1/4$ **D.** $1/4$ E. 1

> **Solution:** We simplify $\frac{x-2}{x^2-4} = \frac{1}{x+2}$ if $x \neq 2$. Thus $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{1}{x+2} = 1/4$.
> Compare WW1.4 #4.

2. (5 points) Let $f(x) = \dfrac{(x+1)(x+2)}{(x+3)(x+4)}$. Find the set of all points x where f is continuous at x.

   A. $(-\infty,-1)\cup(-1,\infty)$
   B. $(-\infty,-4)\cup(-4,-2)\cup(-2,\infty)$
   C. $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$
   D. $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$
   E. $(-\infty,-4)\cup(-4,-3)\cup(-3,\infty)$

> **Solution:** The function f is not defined when $x=-3$ or $x=-4$. Since a rational function is continuous for all points in its domain, the function f is continuous for $(-\infty,-4)\cup(-4,-3)\cup(-3,\infty)$.
> Compare WW1.6 #3.

3. (5 points) Consider the curve defined by the equation $3x^2 + xy + y^2 = 9$. Find the slope of the tangent line to this curve at the point $(1, 2)$.

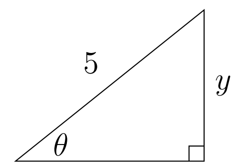
   A. $-5/8$ B. $5/8$ C. $8/5$ **D.** $-8/5$ E. -2

Solution: Implicit differentiation, gives $6x + y + xy' + 2yy' = 0$. Solving for y' gives $y' = -\dfrac{6x + y}{x + 2y}$. Substituting $(x, y) = (1, 2)$ gives $y' = -8/5$.

Compare WS§2.11#3

4. (5 points)

   Consider the right triangle pictured. The length of the hypotenuse is 5 units and is fixed. Find the rate of change of the angle θ with respect to the sidelength y.



   A. $\dfrac{d\theta}{dy} = \dfrac{5}{\sqrt{1 - y^2/25}}$

   B. $\dfrac{d\theta}{dy} = \dfrac{-1}{\sqrt{1 - y^2/25}}$

   C. $\dfrac{d\theta}{dy} = \dfrac{1}{\sqrt{1 - y^2/25}}$

   D. $\dfrac{d\theta}{dy} = \dfrac{-1}{5\sqrt{1 - y^2/25}}$

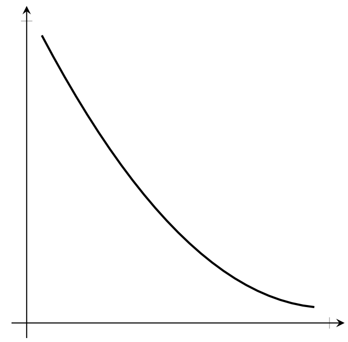
   E. $\dfrac{d\theta}{dy} = \dfrac{1}{5\sqrt{1 - y^2/25}}$

Solution: We have $\theta = \arcsin(y/5)$. Differentiating and using the chain rule gives $d\theta/dy = \frac{1}{5}\frac{1}{\sqrt{1-(y/5)^2}}$.

Compare WS§2.12 #8, WW3.2 #7, Exam 2 #10

5. (5 points)

Consider the function f whose graph appears to the right. Select the correct statement about the derivatives of f.



A. $f'(x) < 0$ **and** $f''(x) > 0$**.**

B. $f'(x) > 0$ and $f''(x) < 0$

C. $f'(x) < 0$ and $f''(x) < 0$

D. $f'(x) > 0$ and $f''(x) > 0$

E. $f'(x) < 0$ and $f''(x) = 0$

Solution: The graph is decreasing and concave up, so $f'(x) < 0$ and $f''(x) > 0$.

Compare WW 3.6.3 #1

6. (5 points) The function P defined by $P(t) = 8e^{3t}$ is growing exponentially. What is the doubling time?

A. $\ln(2)/3$ B. $\ln(3)/2$ C. $\ln(8)/2$ D. $\ln(16)/3$ E. $\ln(2)/8$

Solution: To find the doubling time, we solve the equation $8e^{3T} = 16$ to find $T = \ln(2)/3$.

Compare WW3.3 #2.

7. (5 points) Consider the integral $\displaystyle\int_0^{\pi/2} \sin(x)\,dx$. Give the value of the left Riemann sum formed using three equal subintervals and the left endpoint of each subinterval as the sample points.

   A. $\dfrac{\pi}{12}(1+\sqrt{3})$

   B. $\dfrac{\pi}{6}(1+\sqrt{3}+2)$

   C. $\dfrac{\pi}{6}(1+\sqrt{3})$

   D. $\dfrac{\pi}{3}(1+\sqrt{3}+2)$

   E. $\dfrac{\pi}{12}(1+\sqrt{3}+2)$

Solution: The Riemann sum is $\frac{\pi}{6}(\sin(0)+\sin(\pi/6)+\sin(\pi/3)) = \frac{\pi}{6}(0+\frac{1}{2}+\frac{\sqrt{3}}{2}) = \frac{\pi}{12}(1+\sqrt{3})$.
Compare WW II-1.1#8.

8. (5 points) We have $\displaystyle\int_1^4 f(x)\,dx = M$ and $\displaystyle\int_2^1 f(x)\,dx = N$. Find $\displaystyle\int_2^4 f(x)\,dx$.

   A. $M+N$

   B. M^2+N^2

   C. $-M+N$

   D. $-M-N$

   E. $M-N$

Solution: We have $\int_2^4 f(x)\,dx = \int_2^1 f(x)\,dx + \int_1^4 f(x)\,dx = M+N$.
Compare WW II-1.2#5

9. (5 points) Suppose $\displaystyle\int_{-3}^{4} f(x)\,dx = -8$. Find $\displaystyle\int_{-3}^{4} (2f(x) + x)\,dx$.

A. -10

B. -15

C. $-35/2$

D. $-25/2$

E. $-5/2$

Solution: We have $\int_{-3}^{4} x\,dx = 8 - 9/2 = 7/2$. Thus $\int_{-3}^{4} (2f(x)+x)\,dx = -16 + 7/2 = -25/2$.

Compare WW II-1.2#4. Also, WW II-1.3.

10. (5 points) Let $F(x) = \displaystyle\int_{0}^{x} \sin(\pi t^2)\,dt$. Select the correct statement.

A. $F'(1) = 0$, but F does not have a local extreme at $x = 1$.

B. $F'(1) < 0$

C. F has a local maximum at 1

D. $F'(1) > 0$

E. F has a local minimum at 1

Solution: We use the fundamental theorem of calculus to find the derivative of F is $F'(x) = \sin(\pi x^2)$. At $x = 1$, $F'(1) = \sin(\pi) = 0$, for $0 < x < 1$, $F'(x) > 0$ and for $1 < x < \sqrt{2}$, $F'(x) < 0$. Since F is increasing for $0 < x < 1$ and decreasing for $1 < x < \sqrt{2}$, F has a local maximum at $x = 1$.

Compare WW II-1.3#9 and WW3.6.1-2.

11. (5 points) Find the integral $\int_0^3 |x-1|\,dx$.

Hint: You may find the area of the region between the graph of $y = |x-1|$ and the x-axis.

A. 2 B. 3/2 C. 1/2 D. 1 **E.** 5/2

Solution: The area between the graph of $y = |x-1|$ and the x-axis for $0 \le x \le 1$ is an isosceles right triangle of sidelength 1 and the area for $1 \le x \le 2$ is an isosceles right triangle of sidelength 2. The area is $1/2 + 2 = 5/2$.

Compare WW II-1.3#4 and WW II-1.2 #2.

12. (5 points) Evaluate the integral $\int_0^1 t\sin(\pi t^2)\,dt$.

A. $1/(2\pi)$

B. $-1/\pi$

C. $-1/(2\pi)$

D. 0

E. $1/\pi$

Solution: We substitute $u = \pi t^2$, $du = 2\pi t\,dt$, $u = 0$ when $t = 0$, and $u = \pi$ when $t = 1$. This gives $\int_0^1 t\sin(\pi t^2)\,dt = \frac{1}{2\pi}\int_0^\pi \sin(u)\,du = -\frac{1}{2\pi}(-\cos(u))|_{u=0}^{\pi}$. Evaluating gives $-\frac{1}{2\pi}(-\cos(u))|_{u=0}^{\pi} = \frac{1}{2\pi}(-\cos(\pi) - -\cos(0)) = 1/\pi$.

Compare WW II-1.3 #5, #1.

Free response questions, show all work

13. (10 points) Let $F(x) = -x^3 + 6x^2 + 15x + 11$

 (a) Compute $F'(x)$ and $F''(x)$.

 (b) Use calculus to find the interval(s) on which F is increasing and on which F is decreasing.

 (c) Use calculus to find the interval(s) on which F is concave up and on which F is concave down.

Solution: a) Differentiating, $F'(x) = -3x^2 + 12x + 15 = -3(x^2 - 4x - 5) = -3(x+1)(x-5)$. The second derivative is $F''(x) = -6x + 12$.

b) We have $F'(x) = 0$ if $x = -1$ or $x = 5$. Finding the sign of F', we have

x	$(-\infty, -1)$	-1	$(-1, 5)$	5	$(5, \infty)$
$F'(x)$	$-$	0	$+$	0	$-$
F	$\searrow$		$\nearrow$		$\searrow$

We see that F is decreasing on the intervals $(-\infty, -1]$ and $[5, \infty)$ since $F'(x) < 0$ on $(-\infty, -1)$ and $(5, \infty)$.

The function F is increasing on $[-1, 5]$ since $F'(x) > 0$ on $(-1, 5)$.

c) The function $F''(x) = 0$ if $x = 2$. Finding the intervals where F'' is positive or negative, we have

x	$(-\infty, 2)$	2	$(2, \infty)$
$F''(x)$	$+$	0	$-$
F	$\smile$		$\frown$

Since $F''(x) > 0$ for x in $(-\infty, 2)$, F is concave up on the interval $(-\infty, 2)$. Since $F''(x) < 0$ for x in $(2, \infty)$, F is concave down on the interval $(2, \infty)$.

Compare WW3.6.1-2#3, WW3.6.3#2

Grading summary: [a) 3 points, b) 4 points, c) 3 points]

Grading guidelines: a) First derivative (2 points), second derivative (1 point), b) Finding zeros and sign information for derivative (2 points) Intervals of increase (1 point), intervals of decrease (1 point) c) Zero and sign information for second derivative (2 points), Intervals of concavity (1 point)

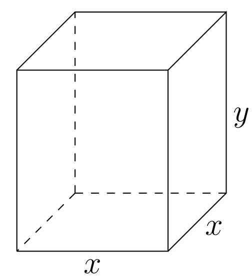
14. (10 points)

We consider a box with a square base and an open top. The box has a volume of 108 cubic centimeters.

(a) Write an expression for the total surface area of the bottom and the four sides in terms of x, the sidelength of the square base.

(b) Find the lengths x and y which give box with smallest surface area.

(c) Use calculus to explain why the sidelength you find in part b) gives the box with the smallest surface area.



Solution: a) The surface area of the box in terms of x and y is $A = x^2 + 4xy$. Using that the volume is $x^2y = 108$, we may eliminate the variable y and obtain that $A(x) = x^2 + 432/x$.

b) We compute the derivative $A'(x) = 2x - 432/x^2$ and solve $A'(x) = 0$ to find the critical point is the solution of $2x^3 = 432$ or $x = \sqrt[3]{216} = 6$. If $x = 6$, then $y = 108/6^2 = 3$. The box with smallest surface area has $x = 6$cm and $y = 3$cm.

c) Studying the sign of A' for $x > 0$, we have

$$\begin{array}{r|ccc} x & (0,6) & 0 & (6,\infty) \\ A'(x) & - & 0 & + \\ A(x) & \searrow & & \nearrow \end{array}$$

Since the function is decreasing on $(0, 6)$ and increasing on $(6, \infty)$, the function A has a global minimum at $x = 6$.

Grading summary: [a) 3 points, b) 6 points, c) 1 point]

a) Expression for area in terms of x and y, (2 points), eliminate y (1 point)

b) Derivative (2 points), solve $A' = 0$ (2 points), determine y (1 point), give lengths of x and y with units (1 point)

c) Explanation (1 point). Students may argue by finding intervals of increase or decrease or observe that $A'' > 0$ for $x > 0$. WW3.5.3#3, WA6.

15. (10 points) (a) Find the indefinite integral $\int \sin(x)\cos(x)\,dx$. Hint: You may use the substitution $u = \sin(x)$.

(b) Find the indefinite integral $\int 4x\sqrt{2x+1}\,dx$. Hint: You may use the substitution $u = 2x + 1$.

Solution: a) If we let $u = \sin(x)$, then $du = \cos(x)\,dx$ or $dx = du/\cos(x)$. Substituting in the indefinite integral

$$\int \sin(x)\cos(x)\,dx = \int u\cos(x)\frac{1}{\cos(x)}\,du = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2(x) + C.$$

b) We substitute $u = 2x + 1$, $x = (u-1)/2$ and $du = 2\,dx$ to give

$$\begin{aligned}\int 4x\sqrt{2x+1}\,dx &= \frac{1}{2}\int 4\frac{u-1}{2}\sqrt{u}\,du = \int u^{3/2} - u^{1/2}\,du \\ &= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2} + C.\end{aligned}$$

Grading summary': [a) 5 points, b) 5 points]

a) Compute derivative for du (1 point), substitute u for $\sin(x)$, (1 point), substitute for dx (1 point), find anti-derivative and write in terms of x (2 points)

b) Compute derivative for du (1 point), substitute u for $2x + 1$ (1 point) substitute $(u-1)/2$ for x (1 point), substitute for dx (1 point), find anti-derivative and write in terms of x (1 point)

Deduct one point if an answer does not include $+C$.

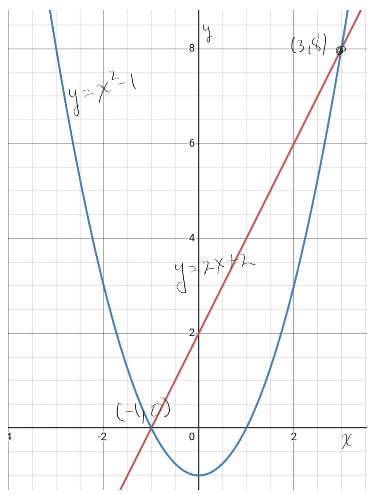
WWII-1.4 #2, #9.

16. (10 points) Consider the region R enclosed by the parabola $y = x^2 - 1$ and the line $y = 2x + 2$.

 (a) Express the area of R as an integral.

 (b) Evaluate the integral from part a) and give the area of R.

Solution:



a) We have the two curves cross when $2x + 2 = x^2 - 1$ or $0 = x^2 - 2x - 3 = (x - 3)(x + 1)$. The solutions are $x = -1$ or $x = 3$. Testing a convenient point such as 0, we see $2 \cdot 0 + 2 > 0^2 - 1$ and thus $2x + 2 \geq x^2 - 1$ for $-1 \leq x \leq 3$ and we have $|2x + 2 - (x^2 - 1)| = -x^2 + 2x + 3$.
The area of R is given as

$$\int_{-1}^{3} |2x + 2 - (x^2 - 1)|\, dx = \int_{-1}^{3} (3 + 2x - x^2)\, dx.$$

b) To evaluate the integral, we find an anti-derivative and use the fundamental theorem to evaluate the definite integral.

$$\int_{-1}^{3} (3 + 2x - x^2)\, dx = (3x + x^2 - \frac{x^3}{3})\Big|_{x=-1}^{3}$$
$$= 9+9-27/3-(-3+1+1/3) = 9+2-1/3 = 32/3.$$

Grading summary: [a) 5 points, b) 5 points]

a) Find points of intersection and give interval of integration (2 points), express area as an integral (3 points) deduct 1 point if integrand is the negative of the correct one.

b) Find anti-derivative (2 points), correct method for evaluation (2 points), answer (1 point).

Compare WWII-1.5 #1.