MA 113 — Calculus I Spring 2009
Exam 4 May 6, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number on the table below. You may also obtain up to 10 extra credit points by correctly answering the five true-false questions on the last page (note that 1 extra credit point will be deducted for each incorrect answer on these questions).

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer for the first ten problems *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		12
2		10
3		7
4		7
5		10
6		9
7		10
8		16
9		16
10		16
EC		10
Free	3	3
		100

(1) Determine the following limits. Justify your steps in finding each limit.

(a) $\displaystyle\lim_{x\to 2} \frac{\sin(3x-6)}{x-2}$, (b) $\displaystyle\lim_{x\to\infty} (\sqrt{x-4}-\sqrt{x+2})$, (c) $\displaystyle\lim_{x\to\infty} e^{-x}\ln x$.

(a) $\displaystyle\lim_{x\to 2} \frac{\sin(3x-6)}{x-2} =$_____, (b) $\displaystyle\lim_{x\to\infty} (\sqrt{x-4}-\sqrt{x+2}) =$_____, (c) $\displaystyle\lim_{x\to\infty} e^{-x}\ln x =$_____

(2) For a constant c, consider the function f defined by

$$f(x) = \begin{cases} \ln(x^2+1)+2 & \text{if } x \leq 0 \\ c\sqrt{x+4} & \text{if } x > 0 \end{cases}.$$

(a) Find c such that f is continuous at $x = 0$. Show all limits that are needed to support your answer.

(b) Use the value for c you found in (a) and determine all points x at which f is differentiable. Indicate your reasoning.

(a) $c =$ ______, (b) f is differentiable on ____________

(3) Suppose that f is a differentiable function, that $f'(x) \geq 2$ for all $x \geq 1$, and that $f(1) = 5$. Use the Mean Value Theorem to argue that $f(x) \geq 2x + 3$ for all $x \geq 1$.

(4) Find the absolute minimum value of $f(x) = x^2 + \dfrac{16}{x^2}$ on the interval $(0, \infty)$. Justify your answer and list also the point(s) where the extremal value occurs.

Absolute minimum value $=$ ____________ at $x =$ ____________

(5) Consider the function $f(x) = \dfrac{x}{x-1}$.

(a) Compute the Riemann sum for f on the interval $[2, 6]$ with $n = 4$ subintervals and the left endpoints as sample points. Give the precise result as a rational number.

(b) Show that f is decreasing on the interval $[2, 6]$.

(c) Without computing the integral $\displaystyle\int_2^6 f(x)dx$, decide whether the Riemann sum in (a) is greater or less than this integral.

(a)Riemann sum = ____________

(c)Riemann sum is ____________ than the integral.

(6) Determine the following integrals. Show your work.

(a) $\displaystyle\int (12x^5 - 4\cos x - 5e^x)dx$, (b) $\displaystyle\int_1^x \frac{t\sin t + 2}{t}dt$, (c) $\displaystyle\int_0^1 x^4 e^{-x^5}\,dx$.

(a) $\displaystyle\int (12x^5 - 4\cos x - 5e^x) =$ ____________________,

(b) $\displaystyle\int_1^x \frac{t\sin t + 2}{t}dt =$ ____________________, (c) $\displaystyle\int_0^1 x^4 e^{-x^5}\,dx =$ __________

(7) A particle is traveling along a straight line so that its velocity at time t is given by $v(t) = 6t - 3t^2$ measured in meters per second.

(a) Sketch the graph of v. Be sure to mark the t-intercepts.

(b) Find the total distance traveled by the particle during the time period $0 \leq t \leq 3$.

Total distance traveled = ________ meters.

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) Define what it means for a function f to be continuous at a. Use complete sentences.

(b) Let c be a number and consider the function

$$f(x) = \begin{cases} \dfrac{\cos x - 1}{x} - c & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ c\ln(e + x) - 4 & \text{if } x > 0 \end{cases}$$

Find all numbers c such that $\lim_{x\to 0} f(x)$ exists.

(c) Is there a number c such that the function f in part (b) is continuous at 0? As always, justify your answer.

(b) $c =$ ____________ (c) yes / no (*circle the correct answer*)

(9) (a) State the Chain rule. Use complete sentences.

(b) Consider the curve described by the equation $\ln(y^2 - 3) = xy - 2$. Find the equation of the tangent line to this curve at the point $(1, 2)$. Write your answer in the form $y = mx + b$. As always, show your work.

(b) Equation of the tangent line is $y =$ ______________

(10) (a) State both parts of the Fundamental Theorem of Calculus. Use complete sentences. Be sure to include the assumptions.

(b) Find the derivative of $F(x) = \int_1^x \cos^3(t)dt$ at $x = \pi$.

(c) Determine the derivative of the function $g(x) = \int_{x^3}^1 \sin^5(t+1)dt$.

(b) $F'(\pi) =$ ____________, (c) $g'(x) =$ ____________________

Extra Credit Problem.
Mark the correct answers below. For each correct answer you earn 2 points, and for each incorrect answer 1 point will be subtracted. Therefore, **it might be wise to skip a question rather than risking losing a point.** However, your final score on this problem will not be negative! You need not justify your answer.

True	**False**	**Statement**
☐	☐	If a function f is increasing then it has an inverse function.
☐	☐	If f is a differentiable function such that $f(1) = -2$ and $f(3) = 4$, then there is a number c such that $f(c) = 0$.
☐	☐	Let f be a function that is defined on a closed interval $[a, b]$. If f is not continuous then f does not have an absolute maximum value on $[a, b]$.
☐	☐	If a function f has a local maximum at a then $f'(a) = 0$.
☐	☐	If f is a continuous function that is even, then the function $F(x) = \int_1^x f(t)dt$ is odd.